MARYLAND STATE DEPARTMENT OF EDUCATION
Achievement Matters Most

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TO: Members of the State Board of Education

FROM: Nancy S. Grasmick

DATE: May 27-28, 2009

SUBJECT: Review of Mathematics Voluntary State Curriculum, Grades K-8

PURPOSE:

The purpose of this item is to report the results of an external review of the Maryland Voluntary State Curriculum pre-kindergarten through grade eight for Mathematics conducted by Achieve, Inc.

BACKGROUND/HISTORICAL PERSPECTIVE:

The No Child Left Behind Act (NCLB) of 2001 requires states to build on the work they had already begun in the area of academic standards by implementing challenging academic content and student achievement standards in the core academic subjects. According to the guidance for NCLB, "The power of rigorous State academic standards is undeniable: they provide a clear direction for what all students should know and be able to do and establish clear expectations for schools, teachers, parents, and students."

Impetus to develop the Voluntary State Curriculum (VSC) resulted from the call for rigorous content standards articulated in NCLB legislation and the 2002 Maryland report, *Achievement Matters Most: The Final Report of the Visionary Panel for Better Schools.* An important recommendation of the Visionary Panel report was for state and local school systems "to align every aspect of education…to support the classroom teacher." This initiative also recommended development of a statewide grade K – 12 curriculum that specifies by grade and subject area what students are expected to know and be able to do.

EXECUTIVE SUMMARY:

The first mathematics VSC was published for pilot use in September 2003 and was accepted by the State Board in June 2004. Maryland is committed to providing teachers with curricular documents that provide clear guidance and establish rigorous expectations for students. Therefore as we approached the five year marker since the implementation

of the Maryland VSC in mathematics, a Request For Proposals (RFP) for expert review was developed. This RFP challenged the respondents to develop a methodology to compare Maryland's Mathematics VSC to the National Mathematics Advisory Panel *Foundations for Success* (NMAP), the National Assessment of Education Progress (NAEP) *Mathematics Framework* 2009—Grades 4 and 8—and the National Council of Teachers of Mathematics (NCTM) *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*. In addition since Achieve, Inc was awarded the contract for the expert review, Achieve's *American Diploma Project (ADP) K-6 and Two-Year Middle School Backmapped Benchmarks for Mathematics* were also used as the basis of comparison for the review of Maryland's *PreK- 8 Voluntary State Curriculum (VSC) in Mathematics*.

The *K-6 and Two-Year Middle School Backmapped Benchmarks for Mathematics* were developed to provide states with the progression of content and skills students would need to master through the grades in order to meet the end-of-high school American Diploma Project (ADP) benchmarks. This extensive work with states on the development of standards has positioned Achieve, Inc. as a leader in the work to develop state-led Common Core State Standards.

The research in mathematics education and the Common Core Standards have provided the context for the latest review of the VSC. The information provided by Achieve has been carefully reviewed by MSDE personnel in anticipation of our next steps.

ACTION:

This item is presented for Board information.

NSG/dls

Attachment : External Review of the Maryland Mathematics Voluntary State Curriculum (VSC)

An Analysis of the *Maryland PreK-8 Voluntary State Curriculum in Mathematics*

A Report submitted by Achieve to the Maryland Department of Education on May 28, 2009

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ACHIEVE'S STANDARDS REVIEW METHODOLOGY

Achieve has been conducting reviews of standards for ten years by benchmarking a state's Academic Standards to "exemplary standards." For purposes of this review, Achieve's *American Diploma Project (ADP) K-6 and Two-Year Middle School Backmapped Benchmarks for Mathematics*, the National Mathematic Advisory Panel *Foundations for Success* (NMAP), the National Assessment of Education Progress (NAEP) *Mathematics Framework* 2009—Grades 4 and 8—and the National Council of Teachers of Mathematics (NCTM) *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* are the standards that were used as the basis of comparison for the review of Maryland's *PreK- 8 Voluntary State Curriculum (VSC) in Mathematics*.

THE DEVELOPMENT OF THE AMERICAN DIPLOMA PROJECT (ADP) K-8 BACKMAPPED STANDARDS IN MATHEMATICS

The American Diploma Project commissioned leading economists to examine labor market projections for the most promising jobs—those that pay enough to support a small family and provide real potential for career advancement—to pinpoint the academic knowledge and skills required for success in those occupations. ADP then surveyed officials from 22 occupations, ranging from manufacturing to financial services, about the skills they believe are most useful for their employees to bring to the job. Following those conversations, ADP worked closely with two- and four-year postsecondary leaders in the partner states to determine the prerequisite English and mathematics knowledge and skills required for success in entry-level, credit-bearing courses in English, mathematics, the sciences, and the humanities. The resulting ADP Benchmarks reflect an unprecedented convergence in what these employers and postsecondary faculty say are needed for new employees and freshmen entering credit-bearing coursework to be successful. Following the creation of the ADP Benchmarks, in 2008 Achieve published on its Web site K-6 standards and model middle school two-year course standards that were back-mapped from and vertically aligned with the ADP Benchmarks to ensure students are prepared to take on the ADP Benchmarks when they enter high school[1].

THE CRITERIA USED FOR THE EVALUATION OF MARYLAND'S VOLUNTARY STATE CURRICULUM IN MATHEMATICS

The development of the ADP Benchmarks, as well as lessons learned over ten years of analyzing state standards; provide Achieve content experts with a comprehensive view of the important characteristics shared by high quality standards. The resulting criteria, which are used in all Quality Reviews and in this evaluation of Maryland's VSC in Mathematics, include **rigor, coherence, focus, specificity, clarity/accessibility, and measurability.**

[1] The ADP Benchmarks in mathematics are currently being updated. After they are published in the fall of 2009, the Backmapped K-6 and Middle School Course Standards will be updated.

The Criteria Used for the Evaluation of Maryland's Voluntary State Curriculum in Mathematics

CRITERIA	DESCRIPTION
Rigor—What is the intellectual demand of the standards?	Rigor is the quintessential hallmark of exemplary standards. It is the measure of how closely a set of standards represents the content and cognitive demand necessary for students to succeed in credit-bearing college courses without remediation and in entry-level, quality high-growth jobs. For Achieve's purposes, the ADP Benchmarks represent the appropriate threshold of rigor.
Coherence—Do the standards convey a unified vision of the discipline, do they establish connections among the major areas of study, and do they show a meaningful progression of content across the grades?	The way in which a state's College and Career Ready Standards are categorized and broken out into supporting strands should reflect a coherent structure of the discipline and/or reveal significant relationships among the strands and how the study of one complements the study of another. If College and Career Ready Standards suggest a progression, that progression should be meaningful and appropriate across the grades or grade spans.
Focus—Have choices been made about what is most important for students to learn, and is at the amount of content manageable?	High quality standards establish priorities about the concepts and skills that should be acquired by graduation from high school. Choices should be based on the knowledge and skills essential for students to succeed in postsecondary education and the world of work. For example, in mathematics choices should exhibit an appropriate balance of conceptual understanding, procedural knowledge and problem solving skills, with an emphasis on application, and in English standards should reflect an appropriate balance between literature and other important areas such as informational text, oral communication, logic, and research. A sharpened focus also helps ensure that the cumulative knowledge and skills students are expected to learn is manageable.
Specificity—Are the standards specific enough to convey the level of performance expected of students?	Quality standards are precise and provide sufficient detail to convey the level of performance expected without being overly prescriptive. Standards that maintain a relatively consistent level of precision ("grain size") are easier to understand and use. Those that are overly broad or vague leave too much open to interpretation, increasing the likelihood that students will be held to different levels of performance, while atomistic standards encourage a checklist approach to teaching and learning that undermines students' overall understanding of the discipline. Also, standards that contain multiple expectations may be hard to translate into specific performances.
Clarity/Accessibility—Are the standards clearly written and presented in an error free, legible, easy-to-use format that is accessible to the general public?	Clarity requires more than just plain and jargon-free prose, which is free of errors. Standards must be communicated in language that can gain widespread acceptance not only by teachers and other educators but also by employers, parents, school boards, legislators, and others who have a stake in schooling. A straightforward, functional format facilitates user access.

CRITERIA	DESCRIPTION
Measurability—Is each standard measurable, observable, or verifiable in some way?	In general, standards should focus on the results, rather than the processes of teaching and learning. The College and Career Ready Standards should make use of performance verbs that call for students to demonstrate knowledge and skills and should avoid using those that refer to learning activities, such as examine, investigate, and explore, or to cognitive processes, such as appreciate.

MAJOR FINDINGS & RECOMMENDATIONS

OVERVIEW

To evaluate the **rigor, coherence, focus, specificity, clarity/accessibility, and measurability** of Maryland's *PreK-8 Voluntary State Curriculum (VSC) in Mathematics*, Achieve reviewers considered the most recent version sent to Achieve in October 2008. Achieve reviewers compared this version with the Achieve *American Diploma Project (ADP) K-6 and Two-Year Middle School Backmapped Benchmarks for Mathematics,* the National Mathematic Advisory Panel *Foundations for Success* (NMAP), the National Assessment of Education Progress (NAEP) *Mathematics Framework* 2009—Grades 4 and 8—and the National Council of Teachers of Mathematics (NCTM) *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*.

Maryland's *PreK-8 VSC in Mathematics* is organized into multiple layers. It is first organized into seven Standards: Knowledge of Algebra, Patterns, and Functions; Knowledge of Geometry; Knowledge of Measurement; Knowledge of Statistics; Knowledge of Probability; Knowledge of Number Relationships and Computation/Arithmetic; and Processes of Mathematics. The seven Standards are then followed by Topics, Indicators, and Objectives. Some Objectives are further described by Assessment Limits.

Since the Objectives and associated Assessment Limits provide the greatest level of detail, these were the primary focus for this review. Achieve was not asked to include in its analysis associated sample problems embedded in the PreK-8 VSC, which exist on Maryland's Web site.

Achieve's major findings from this review are as follows:

- **Maryland's *PreK-8 VSC in Mathematics* is well aligned with the essential procedural content of the *ADP K-6 and Two-Year Middle School Backmapped Benchmarks for Mathematics*.**

  Throughout the major mathematical strands, the *PreK-8 VSC* effectively addresses the procedural skills in mathematics. Most backmapped ADP Benchmarks addressing procedural skills have well aligned counterparts in the *PreK-8 VSC*. For example, in the Grade 5 ADP Number strand, students are expected to "Add fractions with unequal denominators by rewriting them as equivalent fractions with equal denominators." In the *PreK-8 VSC*, students are expected to "Add and subtract proper fractions and mixed numbers with answers in simplest form." Furthermore, through the use of assessment limits, the *PreK-8 VSC* clearly communicates proficiency levels for given Objectives.

- **Attention to focus and coherence will further improve Maryland's *PreK-8 VSC in Mathematics*.**

  In its revision process, Maryland is encouraged to create greater balance in the focus of its document by more clearly addressing the conceptual underpinnings of the content and by increasing the requirement for students to apply their learning by solving complex problems.

To do so, Maryland is encouraged to integrate the Processes of Mathematics Strand into the content strands in a manner that balances application, conceptual understanding (which includes reasoning and connections), and procedural fluency. Because the expectations in this strand are not currently connected to the content strands, these processes may be devalued or over valued in classroom instruction. By integrating Standard 7 with the content strands, Maryland provides districts and teachers with clear guidance as to their relative importance. Furthermore, while they were not considered for this review, Maryland provides sample problems for some Objectives on its Web site. Where these exist, the expectations for conceptual understanding and application of content are clearer. Maryland is encouraged to expand their use, thereby clarifying expectation levels and thus the accessibility of the document.

Regarding coherence, the nature of the *PreK-8 VSC* is to "spiral" or repeat the learning of skills and concepts with a nominal increase in difficulty from year to year. This is especially noticeable in the Number strand. To make the *PreK-8 VSC* more coherent, Maryland is strongly encouraged to set mastery levels for student learning by bringing closure to content in a reasonable timeframe rather than repeating content year after year—ensuring a smooth and logical progression of knowledge and skills across the grades.

- **Overall, there is good alignment between the *PreK-8 VSC* and the ADP Backmapped Benchmarks, the NCTM Focal Points, and the NAEP Objectives in the area of Probability and Statistics, particularly in the earlier grade spans.**

Maryland is to be commended for attending to this content strand in such a thorough manner. This content is not addressed as completely in many states as Maryland has in the *PreK-8 VSC*. For example, in 1.4.A.1, Maryland requires students to "Organize and display data to make line plots using a variety of intervals," is often not addressed until middle school in many states.

FINDINGS AND RECOMMENDATIONS BY CRITERIA

Criterion 1—RIGOR: What is the intellectual demand of Maryland's *PreK-8 VSC in Mathematics*?

Rigor is the quintessential hallmark of exemplary standards; it is a measure of how closely a set of standards represents the content and cognitive demand necessary for students to succeed in the next level of mathematics. For the purposes of this report, four national exemplars represent the appropriate threshold of rigor: the *ADP K-6 and Two-Year Middle School Backmapped Benchmarks for Mathematics,* the National Assessment of Education Progress (NAEP) *Mathematics Framework* 2009, the National Council of Teachers of Mathematics (NCTM) *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*, and the recommendations from the National Mathematics Advisory Panel *Foundations for Success* (NMAP).

❖ ***The Maryland PreK-8 VSC in Mathematics generally defines rigorous expectations around the procedures of mathematics. There are a few gaps that remain between Maryland's Objectives and standards defined in the national exemplars.***

With respect to conceptual understanding and the application of content, Maryland's Objectives are somewhat less rigorous than the national exemplars in all five of the major exemplar strands addressed: Number, Measurement, Algebra, Geometry, and Probability and Statistics. Specific comments on Maryland's Standards follow, with specific commentary organized by ADP strand and exemplar (grade bands K-3, 4-5, and 6-8 are aggregated within the strands):

- **Number**

ADP – An issue with respect to rigor exists in comparing the *PreK-8 VSC* to the ADP K-8 Benchmarks in Number. Since the *PreK-8 VSC* Objectives for Number are primarily skill based, they do not sufficiently address the conceptual understanding associated with number. For example, the *PreK-8 VSC* does not explicitly address the concept of a rational number, the existence of irrational numbers, and properties of numbers and operations—including the properties of addition, additive and multiplicative identities, the associative property of multiplication, properties of zero, and inverse properties, which are fundamentally critical to the later development of algebraic thinking. Although estimation content resides within its own strand under the VSC Standard "Knowledge of Number Relationships and Computation/Arithmetic," rounding and mental calculations are not explicitly addressed, especially when using 10, 100, 1000 in computations. The use of technology is addressed in the description of Standard 6 but not in the specific context of checking for accuracy, which is a critical learning for students. The conceptual generalized model of fractions as lengths on the number line is not expressed. Finally, algorithms are expected to be applied, but there is no evidence that students are expected to explain why they work. Without such an Objective, students may learn the algorithm for its own sake rather than for the sake of intentional application, thus reducing the rigor of the Objective.

NMAP – Overall, the *PreK-8 VSC* addresses the content expected in the NMAP benchmarks. In particular, the *PreK-8 VSC* effectively addresses the efficient use of the addition and

subtraction of whole numbers and fractions. In addition to noting the NMAP benchmarks which do not align well with the *PreK-8 VSC*, Maryland is encouraged to look closely at its Objectives around the comparison of different representations of fractions, decimals, and percents as outlined in NMP.F.1 and NMP.F.2. Currently, the *PreK-8 VSC* does not expect the same level of fluency or conceptual understanding in this area, which is foundationally critical for later development of algebraic thinking.

NAEP – Across Grades K-8, the VSC aligns well with the NAEP objectives in the area of Number Sense. By the end of Grade 4, the *PreK-8 VSC* meets the NAEP objectives in the areas of Number Sense, Numerical Operations, and Properties of Numbers and Operations. In Grades 5-8, the *PreK-8 VSC* aligns with the NAEP objectives in the areas of Number Sense and Ratio and Proportional Reasoning.

Generally, alignment in the area of Estimation is restricted to procedure. Maryland students are expected to estimate the solutions to addition, subtraction, multiplication, and division problems, but they are not expected to know when to estimate or to master common strategies, such as front end rounding. NAEP objectives in the area of Ratio and Proportional Reasoning have few if any counterparts in the *PreK-8 VSC* through the end of Grade 4 only. The same is true in Grades 5-8 in Number Operations and Properties of Numbers and Operations.

Overall, the *PreK-8 VSC* is more focused on the type of numbers used, while NAEP is more focused on what is done with the numbers. For example, the VSC has multiple Objectives for putting different types of numbers on the number line (4.1.C; 5.1.C; 6.1.C ...), while NAEP has standards about representing numbers, decomposing numbers, and connecting various representations of numbers. In its revision process, Maryland is encouraged to focus on Estimation and on how numbers are used.

NCTM – The *PreK-8 VSC* aligns better with the NCTM Focal Points in the earlier grades than it does in later grades. More specifically, the *PreK-8 VSC* has generally good alignment with NCTM's "Number and Operation" and "Number, Operation, and Algebra" strands in Grades K-3. In Grades 4 and 5, alignment continues to be strong in Number and Operations. In several cases, the VSC goes beyond the Focal Points when the focus is on procedural skill.

As was mentioned earlier in this document, the primary departure between the *PreK-8 VSC* and NCTM is in the focus on procedural skill rather than a balanced approach with respect to conceptual understanding. The Focal Points provide for a deeper study of their properties, with a focus on understanding and application. The *PreK-8 VSC* focuses primarily on skill development, as seen in the comparison with Focal Point NO.6-1. NCTM is concerned with understanding the meaning of the numbers and the procedures to solve problems, where the *PreK-8 VSC* limits its focus to skills.

- **Measurement**

ADP – Overall, the *PreK-8 VSC* Objectives through Grade 6 could be more rigorous. While they include an Objective that addresses selecting and using appropriate units, the rigor and

conceptual underpinnings of the backmapped ADP Benchmarks are unclear. For example, where ADP MS1.C1.a calls for students to "Recognize that measurements of physical quantities must include the unit of measurement, that most measurements permit a variety of appropriate units, and that the numerical value of a measurement depends on the choice of unit; apply these ideas when making measurements," the *PreK-8 VSC* requires students to "select and use appropriate tools and units." While it is possible that Maryland's assessments expect students to justify their choices of measurement units, and therefore demonstrate a "recognition that most measurements permit a variety of appropriate units and that the numerical value of a measurement depends on the choice of unit," it is not clear from the Objective alone. In addition, the *PreK-8 VSC* does not address the need for students to understand the relationships among one-, two-, and three-dimensional units of measurement. Precision and accuracy are not included in the context of measurement in the *PreK-8 VSC*. Computation using lengths, weights, capacities, and time are not specifically addressed in the *PreK-8 VSC* measurement Standard at any grade level. Computation with a variety of units is important to the development of future algebraic skills, thus the absence of such Objectives should be addressed.

NMAP – With respect to NMAP, differences in the anticipated grade levels at which the skill should be mastered exist. The *PreK-8 VSC* expects students to master similar content but at a later grade than prescribed by NMAP. For example, in NMP.GM.1, students are expected to solve measurement/geometric problems based on triangles and "all quadrilaterals having at least one pair of parallel sides," by the end of Grade 5. Prior to Grade 5, the only shape the *PreK-8 VSC* requires students to analyze is the rectangle.

NAEP – Two primary themes emerge in the NAEP comparison. First, the two documents align well with respect to the skills of measurement. Second, the alignment is less strong when considering the broader ideas or topics, particularly with respect to number systems. For example, NAEP M.4.1.b asks students to "Compare objects with respect to a given attribute, such as length, area, volume, time, or temperature," while the *PreK-8 VSC* has multiple objectives like, "1.1.B.1.b) Identify and compare units of capacity using cups and gallons; 1.1.B.1.c) Compare and order objects by weight in pounds using a spring scale and a bathroom scale; and 2.1.A.1.e) Identify and compare the weight of objects to the nearest pound." The *PreK-8 VSC* approach could lead to a reductive curriculum that is overly skill based at the expense of conceptual understanding.

NCTM – The *PreK-8 VSC* provides a much more specific and clearer set of standards around Measurement than does the NCTM. Overall, there is good alignment between the two, with a number of *PreK-8 VSC* standards lacking clear counterparts in the NCTM due to their increased specificity.

- **Algebra**

ADP – While most of the *PreK-8 VSC* Objectives in the Algebra strand have counterparts in the backmapped *ADP K-6 and Two-Year Middle School Backmapped Benchmarks,* there is a concern about rigor. The applications of algebra in the *PreK-8 VSC* tend to be at a more

skill-based level, where the application of algebra in the backmapped ADP Benchmarks tends to require a higher level of cognitive demand by the student. For example, in ADP MS2.D3.a, the student must "Recognize that in the form $f(x) = mx + b$, m is the slope, or constant rate of change of the graph of f, that b is the y-intercept and that in many applications of linear functions, b defines the initial state of a situation; express a function in this form when this information is given or needed." Yet, the VSC Objective with the best alignment is 8.1.C.2, which requires students to "Analyze linear relationships: b) Determine the slope of a linear relationship represented numerically or algebraically." The most significant differences between these requirements lies in what students are expected to do with linear relationships. Where the backmapped ADP Benchmark expects students to know how to write a linear function in a specified form when given certain information and to know what the information means, the VSC Objective simply asks students to determine a slope computationally when given numbers or a graph. In other words, where the backmapped ADP Benchmark expects students to take given information, analyze it for its linear properties, and then represent it symbolically as a linear function, the VSC Objective asks students to determine a slope.

The *PreK-8 VSC*'s treatment of the conceptual understandings of algebra needs to be addressed. For example, the *PreK-8 VSC* has no counterpart for ADP MS2.C2.a, which requires students to "Analyze expressions to identify when an expression is the sum of two or more simpler expressions (called terms) or the product of two or more simpler expressions (called factors). Analyze the structure of an algebraic expression and identify the resulting characteristics."

NMAP – The National Mathematics Advisory Panel's *Benchmarks for the Critical Foundations of Algebra* addresses the foundations of Algebra in Number and Geometry. Commentary on the *PreK-8 VSC's* alignment to NMAP can be found in these two sections of this report.

NAEP – Most NAEP objectives have counterparts in the *PreK-8 VSC*, which leads to an overall rating of good alignment. Furthermore, the same issue with respect to cognitive demand raised earlier in this report emerges among those NAEP objectives which have little or no alignment to the *PreK-8 VSC*. For example, where the NAEP objective A.8.4.c requires students to "Analyze situations or solve problems using linear equations and inequalities with rational coefficients symbolically or graphically," the associated VSC standards require students to "Use ratios and unit rates to solve problems," which lacks the application of linear relationships to a contextualized problem.

NCTM – While there are counterparts in the *PreK-8 VSC* to NCTM, thus suggesting generally good alignment, differences exist about the level of cognitive demand required of students by the *PreK-8 VSC* when compared to NCTM. Focal Point A8 provides a good example. The *PreK-8 VSC* has the foundational content that leads to mastery of this Focal Point, but generally the *PreK-8 VSC* content is a precursor to the Focal Point content. More specifically, the VSC references function tables with two-operation rules, while Focal Points discusses special cases of linear equations ($y/x = k$). The first is a very concrete

representation of a linear relationship, while the second is an abstract representation of a specific type of linear relationship, requiring a deeper understanding.

- **Geometry**

ADP – Most of the *PreK-8 VSC* have counterparts in the backmapped ADP Benchmarks in Geometry; therefore, there exists good alignment overall. Those backmapped ADP Benchmarks with little or no alignment have a higher level of cognitive demand than is found in the *PreK-8 VSC*. Cognitive demand is a concern in two areas of geometry—representations of three-dimensional objects in two-dimensional space and in general geometric transformations. Neither area is well addressed in the *PreK-8 VSC*. In cases where they are addressed, the level of cognitive demand is not as high as in the backmapped ADP Benchmarks. For example, where ADP G.5.3.a states that students will be able to, "Represent and work with rectangular prisms by means of orthogonal views, projective views, and nets," the *PreK-8 VSC* counterparts restrain the requirement to, "Compare a plane figure to surfaces of solid geometric figures," (VSC 4.2.B.2.) In these cases, Maryland is encouraged to focus its revision on increasing the cognitive demand of its standards by including greater specificity as to the types of comparisons students should make.

NMAP – The National Mathematics Advisory Panel's *Benchmarks for the Critical Foundations of Algebra* combines Geometry and Measurement. Please see the remarks under Measurement for information on Geometry.

NAEP – Most of the NAEP objectives have counterparts in the *PreK-8 VSC*, which leads to a rating of good alignment. However, of the 17 NAEP objectives with little or no alignment to the *PreK-8 VSC*, the issue of cognitive demand emerges once again. Where the NAEP objectives expect a higher level of cognitive demand, the *PreK-8 VSC* tends to focus on foundational skills. For example, in NAEP G.8.2.d, students are expected to "predict the results of combining, subdividing, and changing shapes of plane figures and solids." The *PreK-8 VSC* counterpart restricts the student requirement to "Estimate and determine area of a composite figure."

NCTM – Overall, there is strong alignment to NCTM with one exception. All Focal Points had good or excellent alignment with the *PreK-8 VSC* except for Focal Point GMA.5. With respect to this comparison, *PreK-8 VSC* Objectives from Grades 4-8 were needed to create an alignment with this Grades 4-5 Focal Point. Because middle school VSC Objectives were found to align with a Grades 4-5 Focal Point, there is a question as to the level of cognitive demand in these VSC Objectives.

- **Probability and Statistics**

ADP – Overall, there is good alignment between the *PreK-8 VSC* and the backmapped ADP Benchmarks in the area of Probability and Statistics, particularly in the earlier grade spans. As the grade level increases, the strength of alignment decreases slightly. For example, while the *PreK-8 VSC* expects students to display data using frequency tables in Grade 6, it is not explicit about the requirement for students to understand or compute relative and

cumulative frequencies. The *PreK-8 VSC* expects students to interpret, organize, and display data using box and whisker plots but does not explicitly require students to find and interpret the median, upper quartile, lower quartile, and interquartile range of a set of data. In order to construct a box and whisker plot, all of these measures must be found. Without explicitly requiring this of students, the question as to how they will create the box and whisker plots emerges. As it is possible to create them with the use of technology, Maryland is encouraged to be more specific as to how the plots should be created.

NMAP – The National Mathematics Advisory Panel's *Benchmarks for the Critical Foundations of Algebra*, to which the *PreK-8 VSC* was compared, does not include a Probability and Statistics strand.

NAEP – Overall, there is strong alignment between NAEP and the *PreK-8 VSC* in this area.

NCTM – All Focal Points had good or excellent alignment with *PreK-8 VSC* counterparts.

Recommendations for improvement:

- Maryland should increase the cognitive demand of its Objectives by focusing on conceptual understanding and problem solving either through a change in Objectives or through an increased use of sample problems that address these skills.

Criterion 2—COHERENCE: Do the content standards convey a unified vision of the discipline, do they establish connections among the major areas of study, and do they show a meaningful progression of content across the grades?

The way in which a set of standards is categorized and broken out into supporting strands should reflect a coherent structure of the discipline and/or reveal significant relationships among the strands and how the study of one complements the study of another. The *PreK-8* VSC should suggest a progression of content that prepares a student for high school.

❖ ***Maryland's PreK-8 VSC generally conveys a unified vision of mathematics and establishes connections among the major areas of study.***

Maryland clearly and explicitly addresses procedural content in the areas of Number, Measurement, Algebra, Geometry, and Probability and Statistics. Less explicit are the expectations that address the conceptual underpinning of each Objective. In Number, as noted earlier, the *PreK-8 VSC* does not explicitly address the concept of a rational number, the existence of irrational numbers, and properties of numbers and operations—including the properties of addition, additive and multiplicative identities, the associative property of multiplication, properties of zero, and inverse properties. However, Maryland has created a set of standards that evidences some classic areas of integration or connection among the mathematical strands. These connections include counting and pattern recognition (number-algebra) and analytic geometry. All in all, the *PreK-8 VSC* does a strong job of presenting a unified vision of mathematics and establishing connections among the major areas of study.

❖ ***In some areas, Maryland's PreK-8 VSC repeats Objectives across grade levels with only minor changes rather than defining and developing a progression of content and skills across PreK-8.***

The organizational structure of the Maryland standards suggests a repetitive structure for conveying the progression of content and skills in mathematics. While grade level based proficiency is apparent in the assessment limits, there is no clear communication of when students are expected to have mastered particular content. Rather, topics are continued from one grade to the next without sufficient closure. The Plane Geometry strand in Standard 2 provides an illustration of repetition rather than significant progression. For example, VSC 3.2.A.1.a, 4.2.A.1.a, 5.2.A.1.a, and 6.2.A.1.a carry on the identification and description of various lines and line segments across four years.

Within VSC Standard 1—Knowledge of Algebra, Patterns, and Functions—there are only a few Objectives which repeat where the backmapped ADP Benchmarks do not. The following VSC Objectives repeat and thus have no counterparts in the backmapped ADP Benchmarks, as the backmapped ADP Benchmarks do not continue benchmarks focused on patterns after Grade 3. Beginning in Grade 4, students must represent and analyze numeric patterns of 3, 4, 6, 7, 8, or 9 starting with any whole number 0-100 using skip counting. Students must also create one-operation (+ or –) function table to solve a real world problem, complete a function table using a one-operation (+,–, x, / with no remainders) rule for whole numbers 0-50, and describe the relationship that generates a one-operation rule. Also in Grade 4, students must generate a rule for the next level of the growing pattern using at least three levels but no more than five levels, generate a rule for a repeating pattern using no more than four objectives in the core of the

pattern, and create a non-numeric growing or repeating pattern. In Grade 5, students must create a one-operation (x, / with no remainders) function table to solve a real world problem, complete a one-operation function table using whole numbers with +, –, x, / (with no remainders) or use decimals with no more than two decimals places with +, – for numbers 0-200. Finally, they must apply a given two-operation rule for a pattern using two operations (+, –, x) and whole numbers 0-100.

With respect to Probability, the *PreK-8 VSC* introduces content earlier than the backmapped ADP Benchmarks. Where ADP does not address probability in Grades K-3, the VSC does, with Objectives 1.5.A.1.a, 2.5.A.1.a, 3.5.A.1.a, 3.5.A.1.b, 3.5.B.1.a. Some *PreK-8 VSC* Objectives from Grades K-5 align with backmapped ADP Benchmarks for middle school. They are 5.5.A.1.a, 4.5.B.1.a, and 5.5.B.1.a 7. The inclusion of these topics at earlier grades does not increase the rigor of the *PreK-8 VSC* but does suggest a slightly different order of when students are first introduced to these topics.

Recommendations for improvement:

- Maryland should consider including in the *PreK-8 VSC* the knowledge and skills related to Number that have been identified in this report to create a more unified vision of mathematics.
- A progression of knowledge and skills should be defined and developed coherently and brought to closure as early as possible across Grades PreK-8, rather than repeating Objectives, with only minor variation, from one grade level to the next.

Criterion 3—FOCUS: Have choices been made about what is most important for students to learn, and is the amount of content manageable?

High quality standards establish priorities about the concepts and skills that should be acquired before entering high school. Choices should be based on the knowledge and skills essential for students to advance to the next level of understanding. In mathematics, choices should exhibit an appropriate balance of conceptual understanding, procedural knowledge, and problem solving skills—with an emphasis on application. A sharpened focus also helps ensure that the cumulative knowledge and skills students are expected to learn is manageable.

❖ ***Maryland's PreK-8 VSC is generally manageable.***

The total body of expectations would most likely be addressed in an instructional program spanning Grades PreK-8. Some students will be able to complete these standards in earlier grades and should have accelerated options for the final years before high school mathematics study or be allowed to begin study of high school mathematics early. With this set of standards mastered, a student would be prepared for the procedural skills of an Algebra I course.

❖ ***Maryland's PreK-8 VSC could more effectively balance conceptual understanding, procedural fluency, and problem solving.***

As noted previously in this analysis, there is a significant gap around the issue of balance. Where the backmapped ADP Benchmarks require students to learn conceptual understandings, procedural skills, and to apply their learning, the *PreK-8 VSC's* focus lies almost entirely in the realm of procedural skill. In each of the content strands, Objectives are consistently lacking which require students to understand underlying concepts and apply knowledge. In the Communication strand of the process standards, students are expected to "use multiple representations to express mathematical concepts using concrete, pictorial, tabular, graphical, and algebraic methods." This standard could be the vehicle for expressing the importance of conceptual understanding were it more directly connected with the content standards. VSC Standard 7—Processes of Mathematics—provides a focus on reasoning and problem solving. Yet, without integration into the content standards, teachers and districts may over- or under-emphasize them.

Recommendations for improvement:

- Maryland may want to consider whether it wants to broaden its Grades PreK-8 Objectives to meet the needs of students who progress more quickly through the content or allow students to begin studying high school content prior to high school.
- Maryland should integrate its Standard 7 Objectives within Standards 1-6 to create a better balance among conceptual understanding, procedural skills, and problem solving. While the existence of sample problems on its Web site increases the focus of the PreK-8 VSC, Maryland should either significantly increase their number or change the language of the Objectives to incorporate Standard 7.