5

HIGH SCHOOL ASSESSMENT

MATHEMATICS

CORE LEARNING GOALS

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PREFACE

The Maryland School Performance Program began in 1989 in response to the report of the Governor's Commission on School Performance and reflects a major strategy for implementing school reform to improve educational opportunity and achievement by each student enrolled in Maryland's public schools. The high school assessment represents the final stage of the Maryland School Performance Assessment Program, which began with State-level assessments in grades 3, 5, and 8.

This document reflects the work of five content teams, appointed by the State Superintendent of Schools, to define Core Learning Goals that will serve as the basis of the assessment. This work is a direct outgrowth of the State Board of Education's Performance-Based Graduation Requirements Task Force.

The outcomes were prepared by a representative group of educators, recognized for their leadership in the fields of English, mathematics, social studies, and science. The Skills for Success component represents a cooperative effort between leading educators and the Maryland Business Round Table. The Core Learning Goals are meant to reflect the essential skills and knowledge that should be expected of Maryland high school students in the 21st century. There is no assumption that the State's high schools currently have the capacity to deliver these goals. Rather, if the goals are adopted, an infrastructure of support and professional development activities, including human and fiscal resources, will be necessary to implement these new standards. Each of the five documents is available upon request to the address listed below.

It is important to note that the Core Learning Goals for Skills For Success are meant to be part of each of the other four content areas. As such they will not be assessed by their own test, but rather within each of the four content areas. The test materials in each area will be developed in such a way that mastery of the Skills for Success is essential to high performance. Hence, you will notice that each of the documents has a section related to Skills For Success. It is also our intent that all teachers, not just those who are teaching English, mathematics, social studies, and science, will be responsible for Skills for Success. It will be important, therefore, that the Skills for Success document is shared with all high school teachers. The graphic that follows is intended to show the relationship between and among the content area and Skills for Success.

The content area information is provided as draft material representing the best thinking of the content teams for public consideration by educators and the public at large. The Content Team membership list is included as an appendix. Individuals and organizations may feel free to duplicate and disseminate the document as appropriate. It is also assumed that prior to adoption by the State Board of Education, or to curriculum redesign occurring at the local school system level, these documents should be shared with the appropriate departments in each high school in Maryland. Information should be gathered as to how departments are interpreting the goals, in order that the content teams may review the diversity of interpretations. Upon review of the anticipated diversity of responses, each Core Learning Goals Content Team would identify the level of specificity for the goals that clearly identifies the intent. At that point the outcomes would be published in the *Maryland Register* in preparation for State Board adoption.

Responses, reactions, and comments may be sent by mail or by fax to:

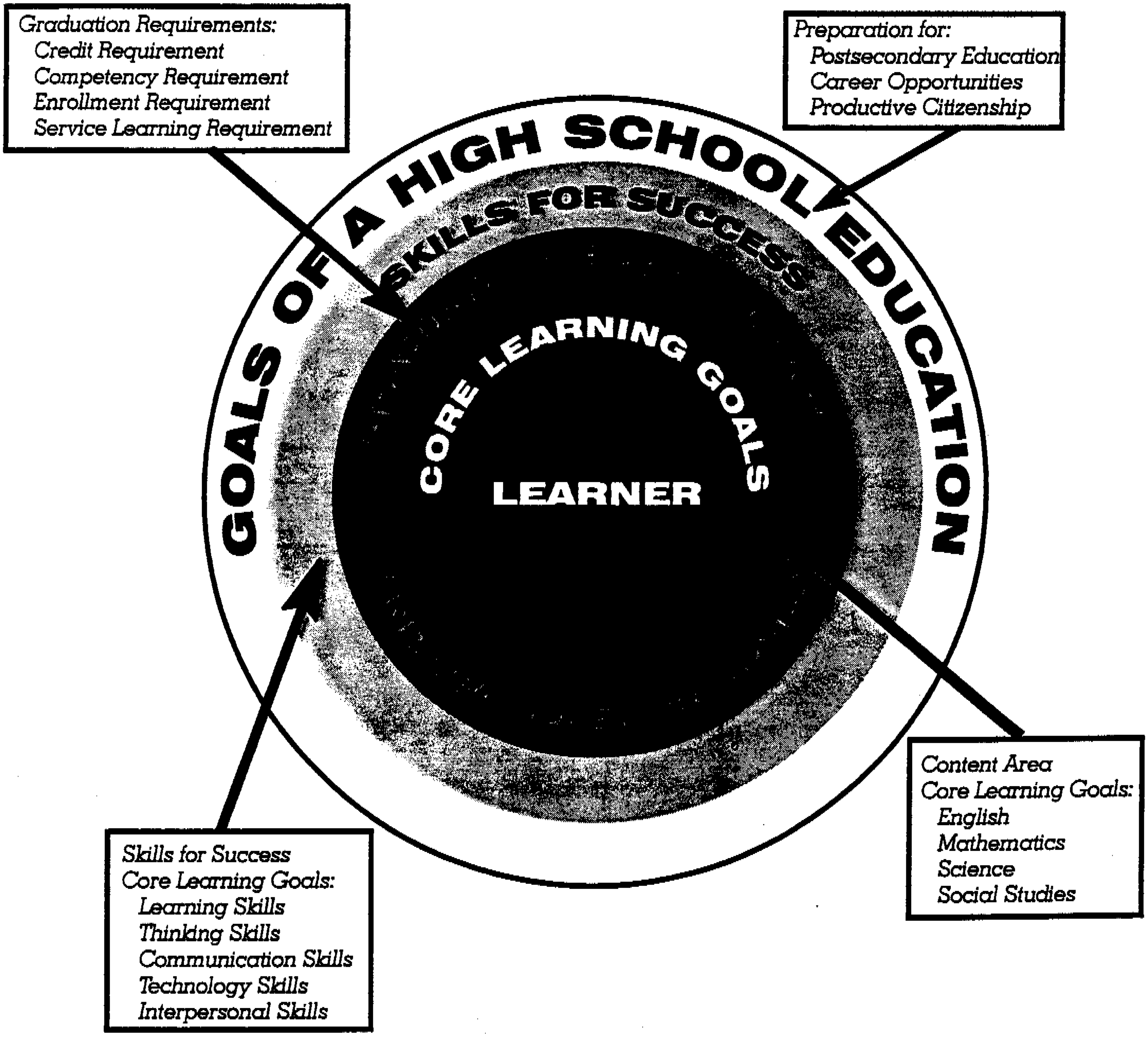
Robert E. Gabrys, Chair
High School Assessment Task Force
Maryland State Department of Education
200 West Baltimore Street
Baltimore, Maryland 21201
Fax: (410) 333-3867

Addresses are provided for each of the Content Team members at the end of the document. Any individual should feel free to discuss issues with these individuals. MSDE staff in the content area are also available for explanation of the Core Learning Goal documents.

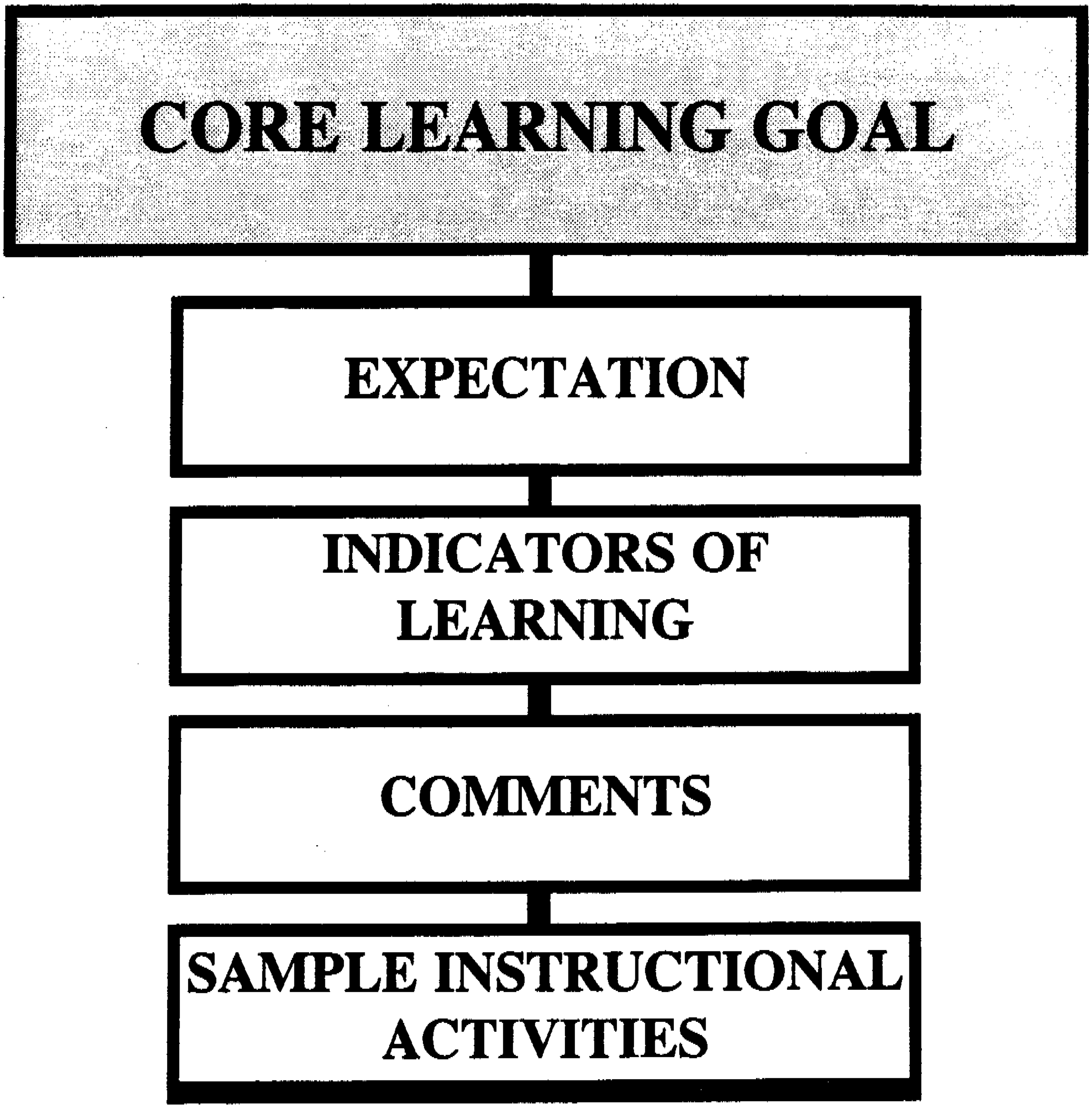
Thank you in advance for your interest and willingness to aid in the development of high-quality expectations for Maryland high school students prior to graduation.

July 3, 1995
preface.reg

CONTEXT OF THE HIGH SCHOOL ASSESSMENT



HIGH SCHOOL ASSESSMENT
CORE LEARNING GOALS FORMAT
MATHEMATICS



MATHEMATICS
CORE LEARNING GOALS

Introduction/Rationale

All students of the 21st century must be mathematically literate to perform in the workplace, be lifelong learners, and be confident problem solvers. Our changing society demands a working knowledge of patterns, functions, algebra, spatial relationships, geometry, measurement, data analysis, probability, and competent use of technology. As our society and technology changes, so too does the mathematics which is accessible to students and the way in which it is taught and learned.

In keeping with the Curriculum and Evaluation Standards for School Mathematics from the National Council of Teachers of Mathematics, traditional topics of algebra and geometry remain important components of the secondary school mathematics curriculum. However, the core learning goals call for a shift in emphasis from memorization of isolated facts and procedures and proficiency with paper-pencil skills to emphasis on conceptual understandings, multiple representations and connections, mathematical modeling, and mathematical problem solving. The integration of ideas from algebra and geometry is particularly strong, with graphical representation playing an important connecting role. Reference is made in the expectations to the use of current technology which includes computers and calculators as appropriate. In addition, topics from data analysis and probability are elevated to a more central position in the core learning goals for all students.

The elements of this core content, compiled to drive mathematics instruction in the high school, are not meant to be separate mathematical courses taught in any particular sequence. This content assumes mathematical preparation at the K-8 level consistent with Maryland Mathematics Outcomes. Processes such as problem solving, communication, reasoning, and connections should be embedded throughout the content and provide practice of the Skills for Success. Real-world applications are the backbone of this content and show students the inherent value and power of mathematics.

Technology is vital to the study of mathematics. While today that technology takes the form of computers and graphing calculators, it is essential that technologies continue to reflect current standards. With the change in technologies, the mathematical processes change.

All students will have equal access to this highly challenging, 21st century mathematics content. All students are expected to demonstrate the ability to apply mathematical knowledge and problem-solving strategies to numerous activities through high school mathematics.

MATHEMATICS
CORE LEARNING GOALS

GOAL 1: FUNCTIONS AND ALGEBRA

The student will demonstrate the ability to investigate, interpret, and communicate solutions to mathematical and real-world problems using patterns, functions, and algebra.

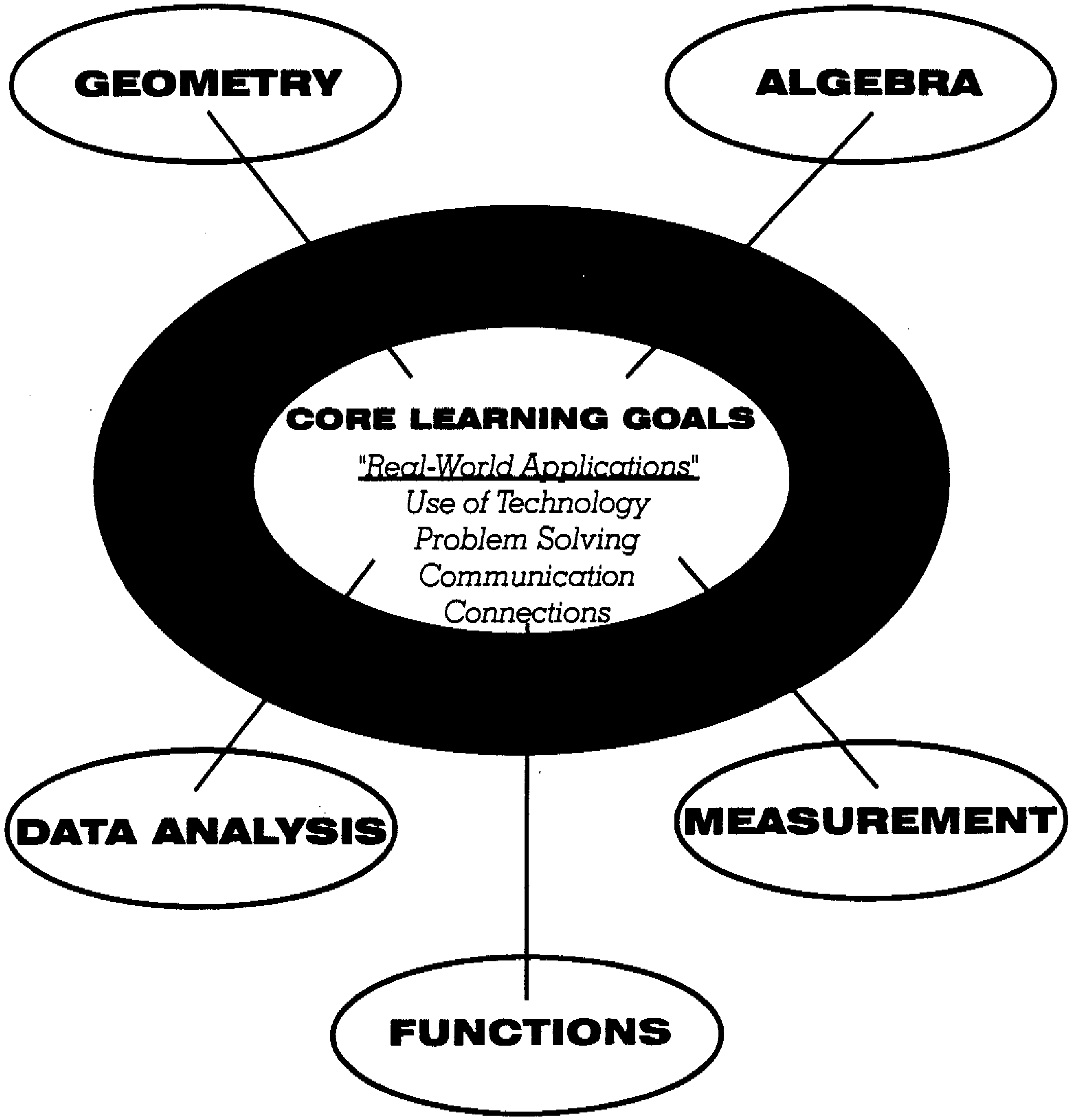
GOAL 2: GEOMETRY, MEASUREMENT, AND REASONING

The student will demonstrate the ability to solve mathematical and real-world problems using measurement and geometric models and will justify solutions and explain processes used.

GOAL 3: DATA ANALYSIS AND PROBABILITY

The student will demonstrate the ability to apply appropriate technologies and statistical methods for representing and interpreting data and communicating results.

MATHEMATICS CONCEPTS AND THEIR RELATIONSHIP TO THE CORE LEARNING GOALS



GOAL 1: FUNCTIONS AND ALGEBRA

The student will demonstrate the ability to investigate, interpret, and communicate solutions to mathematical and real-world problems using patterns, functions, and algebra.

1.1 ***Expectation:*** ***The student will analyze a wide variety of patterns and functional relationships using the language of mathematics and appropriate technology.***

Indicators of Learning

1.1.1 The student will recognize and describe patterns and sequences that are expressed numerically, algebraically, and geometically.

1.1.2 The student will extend and interpolate pattern and functional relationships.

1.1.3 The student will represent patterns and functional relationships in a table, as a graph, and/or by mathematical expression.

1.1.4 The student will add, subtract, multiply, and divide algebraic expressions, use matrices (arrays of numbers), and apply formulas to solve real-world problems.

Comments

The following comments are applicable to *every* Expectation:

- Instruction at all levels should include opportunities for students to read, interpret, and apply mathematics in context from textbooks and appropriate materials in a variety of ways. Assignments that require students to read mathematics and respond both orally and in writing to questions based on their reading should be an integral part of the mathematics program.
- Multiple approaches to attain the indicators are encouraged.
- Examples are provided to encourage the consideration of a variety of instructional strategies and the use of a variety of tools and resources.
- The examples will try to use authentic situations. This implies the vocabulary used must be clarified as part of instruction.
- Examples are illustrative of the intent of the indicators but are not all-inclusive.

The following comments are specific to Expectation 1.1:

- Patterns will be used as a major tool in understanding mathematical relationships.
- Patterns in geometry may include three-dimensional figures.
- Matrices will be addressed through manipulation of data in tables.

Examples

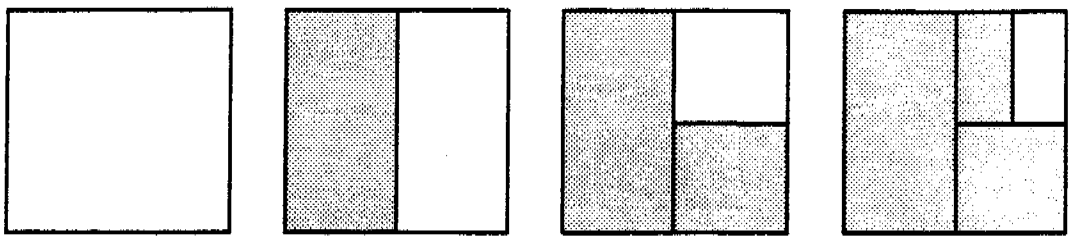
1. **Indicators of Learning 1.1.1, 1.1.2, 1.1.3**

The temperature of the air influences the number of chirps a cricket makes. By counting the number of chirps in a minute, you can determine the temperature. Look at the chart below.

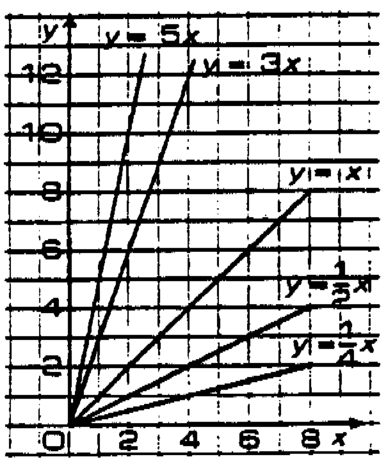
Chirps	40	60	80
Temperature	14°C	18°C	22°C

1. Predict the temperature when the number of chirps is 100.
2. When would this problem have no meaning?

2. **Indicator 1.1.1**



Draw the next figure.

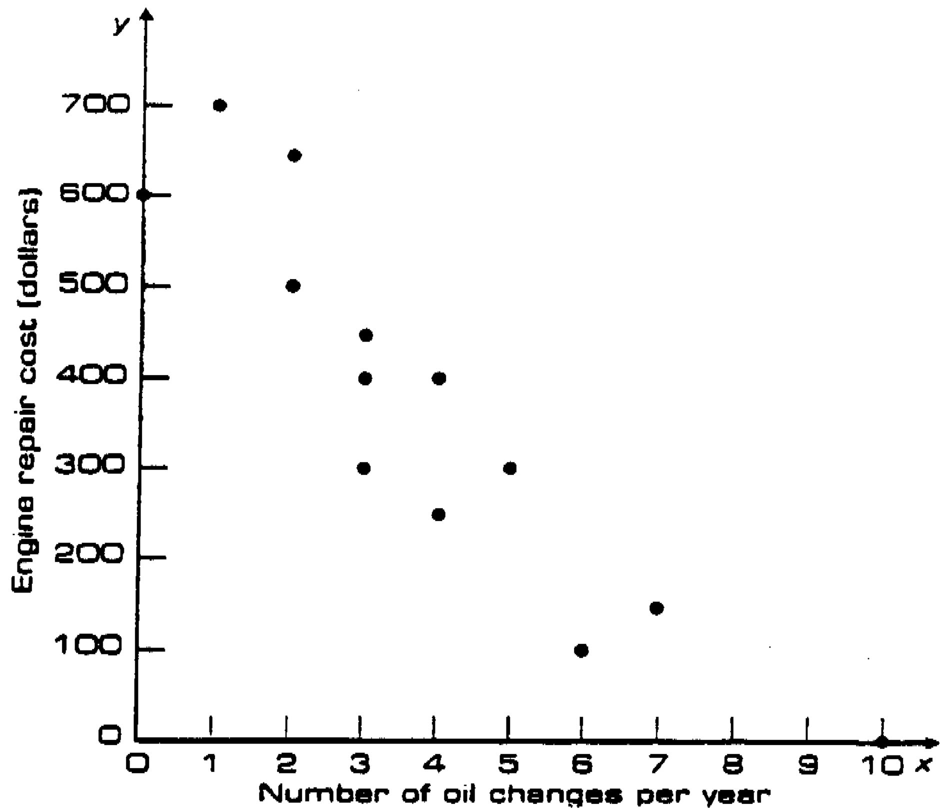
3. 

Indicators of Learning 1.1.1, 1.2.1

1. Explain the similarities and differences among the equations and the lines.
2 Explain how the differences in the equations is reflected by the graphs.
3. Sketch on the graph where you would expect to find the line y = 2x.

Source: A Core Curriculum, Addenda Series, Grades 9-12. Reston: National Council of Teachers of Mathematics, 1992. Reprinted with permission.

4. **Indicator 1.1.1**

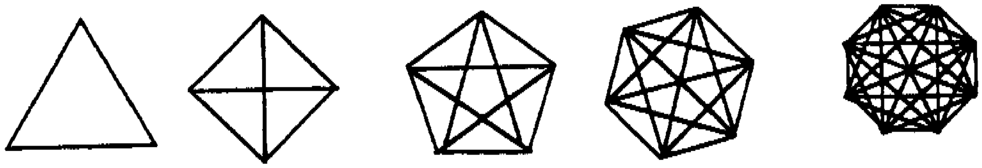


1. Describe the relationship between the number of oil changes and the cost of car repair.
2. Write an appropriate title for this graph.

Source: Data Analysis and Statistics, Addenda Series, Grades 9-12. Reston: National Council of Teachers of Mathematics, 1992. Used with permission.

5. **Indicator 1.1.1**

Look at the regular polygons below. Each one has all its diagonals drawn.



Number of Sides	3	4	5	6	7	8
Number of Diagonals						

1. Fill in the table for the given polygons.
2. Notice that a pattern is developing. Explain the pattern.
3. Without drawing, determine how many diagonals a 7-sided polygon will have.

6. **Indicators of Learning 1.1.1, 1.1.2, 1.1.3**

Look at the following pattern.

4
4+5+4
4+5+6+5+4
4+5+6+7+6+5+4

1. Explain the pattern.
2. What would you expect the sixth row to be?
3. What do you notice about the sum of each row?
4. What would you expect the sum of the sixth row to be?

7. Indicator 1.1.3

Four players threw a pair of dice twice. On the first round of throws, Jeff's score is represented by f. Martha scored 4 more than Jeff. Quincy scored 2 less than Jeff, and Kwan scored 1 less than Martha. Use the diagram below to summarize the information for the first round to help you answer the following questions.

	First Round	Second Round	Total for Both Rounds
Jeff			
Martha			
Quincy			
Kwan			

1. List the possible scores for Jeff.
2. List the possible scores for Quincy.
3. How many more points did Martha score than Quincy?
4. If Jeff and Kwan were teammates against Martha and Quincy, which team scored the most points on the first round?

On the second round, Jeff's score is represented by s. Martha scored 3 less than Jeff. Quincy scored 2 more than Jeff. Kwan scored 2 less than Quincy. Use this additional information to complete the diagram above, and then answer these questions:

5. Which players received the same score on the second round?
6. List the possible scores that Jeff could have scored on the second round.
7. Who scored the most total points for the two rounds?
8. If Quincy scored a total of 15 points, list the total points for each of the other players.

Source: A Core Curriculum, Addenda Series, Grades 9-12. Reston: National Council of Teachers of Mathematics, 1992. Used with permission.

8. Indicator 1.1.4

The Cuddly Toy Company manufactures two types of stuffed animals: pandas and kangaroos. The production of each toy requires cutting materials, sewing, and finishing. This matrix shows the number of hours of each type of labor required for each type of toy.

Labor Matrix

	Panda	Kangaroo
Cutting	0.5	0.8
Sewing	0.8	1.0
Finishing	0.6	0.4

The company has received orders for the months of October and November. This matrix shows the number of each type of toy to be produced each month.

Order Matrix

	October	November
Panda	1000	1100
Kangaroo	600	850

We need to calculate the total hours for both October and November. The labor matrix and the order matrix are given above. Calculate and fill in the October and November totals.

Hours Matrix

	October	November
Cutting		
Sewing		
Finishing		

Source: Connecting Mathematics, Addenda Series, Grades 9-12. Reston: National Council of Teachers of Mathematics of Mathematics, 1991. Used with permission.

1.2 ***Expectation**: The student will model and interpret real-world situations, using the language of mathematics and appropriate technology.*

Indicators of Learning

1.2.1 The students will determine the equation for a line, solve linear equations, and describe the solutions using numbers, symbols, and graphs.

1.2.2 The student will determine if two straight lines intrsect and, when possible, describe the inteersection using numbers, symbols, and graphs.

1.2.3 The student will solve and describe if and where two straight lines intersect using numbers, symbols, and graphs.

1.2.4 The student will describe the graph of a function which is not a straight line and discuss its appearance in terms of maxima and minima (highs and low), roots (zeros), limits (boundaries), rate of change, and continuity.

Comments

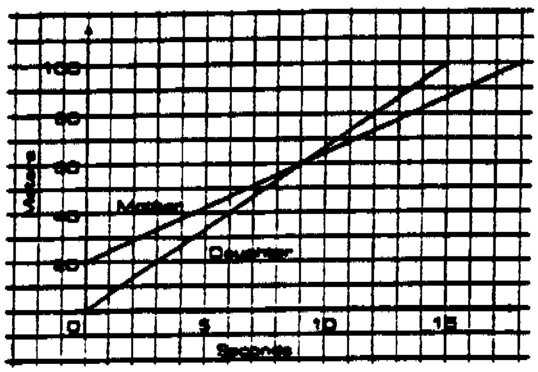
The following comments are specific to Expectation 1.2:

- Functions are to have no more than two variables with rational coefficients.
- Functions may include step, absolute value, or piece-wise functions.
- Either form, $Ax+By=C$ or $y=mx+b$, is acceptable for linear functions.
- Systems of linear functions may include the same, parallel, or intersecting lines.
- For non-linear functions, the emphasis will be on relationships between applications and mathematical models. Students will be asked to describe how the model represents the problem and to estimate solutions.

Examples

1. Indicators of Learning 1.1.1, 1.2.3

The following graphs represent the progress of a 100-meter race by a mother against her daughter (assuming that each ran at a constant rate). The mother agreed to race only if she were given a 20-meter head start. The graphs give the distance from the starting line of each contestant t seconds after the start. Answer the following questions from the information given by the graph.

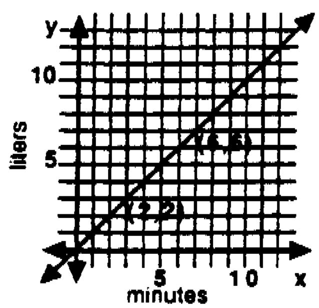


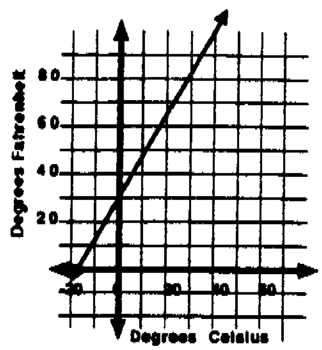
1. How far from the starting line was each runner after 2 seconds?
2. At what time did each runner reach the 40-meter mark?
3. If the equation for the daughter's graph is $d = 6.67t$ and the equation for the mother's graph is $d = 4.5t + 20$, what does the point of intersection of the two graphs mean? When did it occur? At what location?

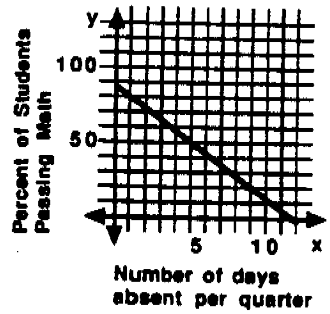
Source: A Core Curriculum - Making Mathematics Count for Everyone: Addenda Series, Grades 9-12. Reston: National Council of Teachers of Mathematics, 1992. Reprinted with permission.

2. **Indicator 1.2.1**

Write an equation in words that shows the quantity on the vertical axis related to the quantity on the horizontal axis.







3. **Indicators of Learning 1.2.1, 1.2.3**

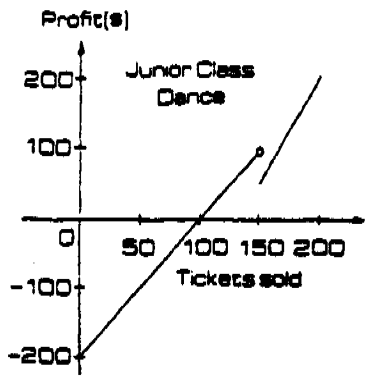
A bank claimed that its new check service will save the customer money. The cost of the old plan was a \$4.00 monthly fee and \$.10 per check. The new plan requires a \$5.00 monthly fee and costs \$.06 per check. How many checks must a customer write per month before the new service is better for the customer?

4. **Indicator 1.2.2**

John goes to an amusement park. He has \$30. If the admission price is \$8.00 and each ride costs \$.50, how many rides can he go on? Write and graph a mathematical sentence to model this problem. What is the domain?

5. Indicator 1.2.4

The Junior class treasurer presented the following graph to the dance committee.

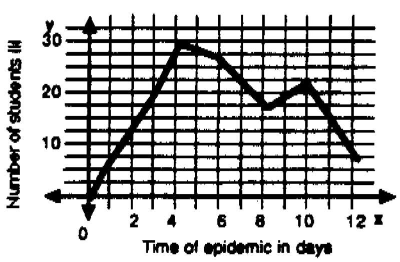


1. What does the ordered pair (0,-200) mean?
2. How many tickets were sold before breaking even?
3. A security officer is required if more than 150 tickets are sold; how is this reflected in the graph?

Source: Curriculum and Evaluation Standards for School Mathematics. Reston: National Council of Teachers of Mathematics, 1989. Used with permission.

6. Indicator 1.2.4

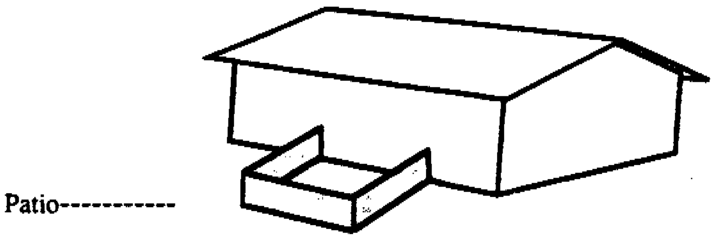
When a measles epidemic starts in a school, the number of affected students varies over the next several weeks. The graph below illustrates the most recent outbreak.



1. Over which days was the spread of the epidemic increasing? Decreasing?
2. The graph appears to have two high points. Where are they and what do they mean?
3. We also see several low values. Where do these occur and what do they mean?

7. Indicators of Learning 1.2.4, 1.1.3, 2.1.2, 2.4.3

The high school wants to build a patio adjacent to the cafeteria. Fencing will be used around three sides of the patio and the cafeteria will be the fourth side of the patio. There is 50 feet of fencing. Find the dimensions that give the maximum area. The area of a rectangular region is length times width.



1. Draw and label a diagram.
2. Write an equation for area as a function of width.
3. Graph the equation with a graphing tool.
4. What part of the graph represents the problem situation?
5. Use the graph to estimate the width of the patio for the maximum area.
6. Estimate the maximum area.

GOAL 2: GEOMETRY, MEASUREMENT, AND REASONING

The student will demonstrate the ability to solve mathematical and real-world problems using measurement and geometric models and will justify solutions and explain processes used.

*2.1 <u>Expectation</u>: **The student will represent and analyze two- and three-dimensional figures using tools and technology when appropriate.***

<u>Indicators of Learning</u>

2.1.1 The student will describe the characteristics of geometric figures in oral and written form.

2.1.2 After hearing or reading a description, the student will construct or draw geometric figures using technology and tools.

2.1.3 Using the coordinate plane, the student will investigate properties of geometric figures using concepts from algebra.

2.1.4 The student will use transformations to move figures, create designs, and understand geometric properties.

<u>Comments</u>

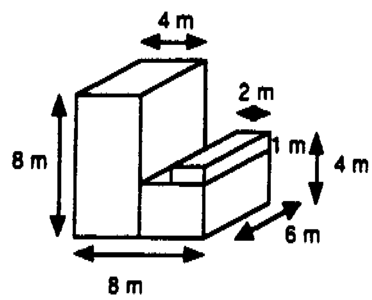
The following comments are specific to Expectation 2.1:

- When using the coordinate plane, concepts from algebra include use of the distance, midpoint, and slope formulas.
- Transformations include reflections, rotations, translations, and dilations.
- 'Tools' refers to ruler, compass, and protractor; 'technology' refers to computer software and graphing calculators.

Examples

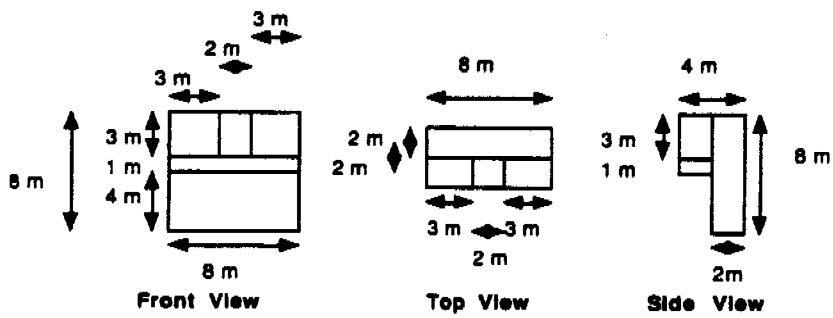
1. Indicator 2.1.1

For the figure below, write a description using geometry so that someone could draw the figure without first seeing it.



2. Indicator 2.1.1

Describe the three-dimensional figure represented by these three views.

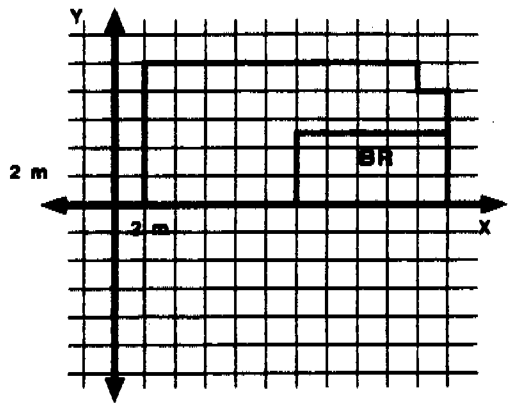


3. Indicators of Learning 2.1.2, 2.1.3, 2.3.2

Two campers want to sight bald eagles. They have been told that there are two, one due east, and one due north. The campers hike in these directions and sight eagles in trees at the points B=(6,0) and C=(0,14). They decide to set up camp overnight halfway between B and C. Using the proper coordinates, explain where they set up camp. Justify your answer.

4. **Indicators of Learning 2.1.3, 2.1.4**

An architect is designing rooms for a motel to minimize cost of materials. The shape of a room is shown below on a coordinate plane, where **BR** is the location of the bathroom. The architect is looking for the best possible way to minimize the amount of plumbing pipes needed. She realizes that reflecting the room over the x-axis gives a clear picture of this idea.



1. Label the coordinates of all vertices of the figure.
2. Sketch the reflection of each figure over the x-axis.
3. Label the coordinates of the vertices of the reflected figure, the image.
4. What is the relationship of a point and its image?
5. Explain how this configuration minimized the amount of plumbing pipes.

2.2 ***Expectation*****:** ***The student will apply geometric properties and relationships using tools and technology when appropriate.***

Indicators of Learning

2.2.1 The student will investigate congruent and similar figures and apply equality or proportionality of their corresponding parts.

2.2.2 The student will solve problems using two-dimensional figures and/or right-triangle trigonometry.

Comments

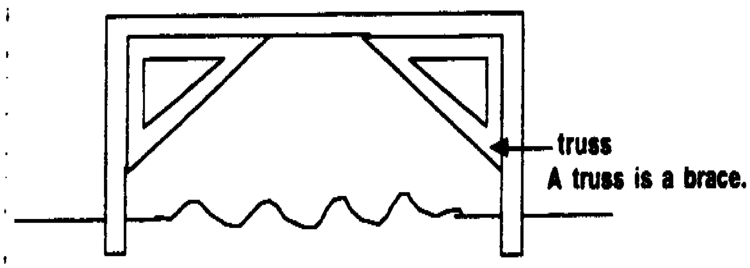
The following comment is specific to Expectation 2.2:

- Trigonometric functions will be limited to sine, cosine, and tangent.

Examples

1. Indicators of Learning 2.2.1, 2.3.1

A bridge is being constructed. Before two large triangular trusses can be hoisted into place, the contractor has to verify that the trusses are identical. What are the fewest number of parts which must be measured? What parts? Describe one way to verify that the trusses are identical using the least number of parts.



2. Indicator 2.2.2, 2.3.2

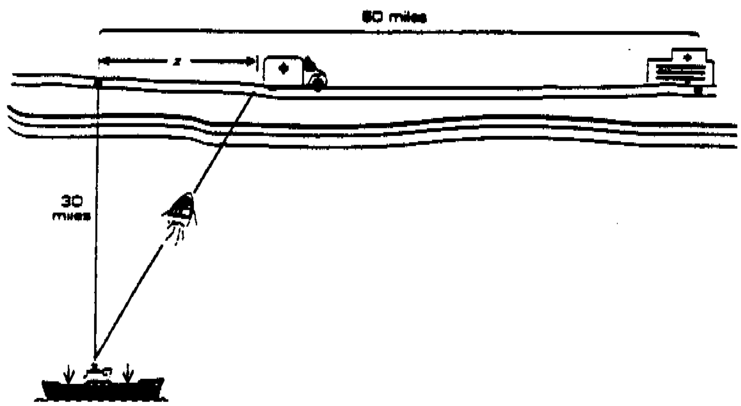
A football player ran 12 yards in one direction and then 5 yards in another direction. Which of the following could be the player's distance from his starting point? Justify your answer.

a. 17 yards b. 10 yards c. 18 yards d. 6 yards

3. **Indicators of Learning 2.2.2, 1.1.4**

You are the captain of a ship, and one of your passengers has been injured. Your ship is 30 miles from a point that is 60 miles downshore from a hospital. You must order an ambulance to meet your ship at any point along a road that runs parallel to the shoreline. You would like to meet the ambulance at a point that will get your passenger to the hospital in the shortest possible time. Suppose that your boat travels at a rate of 20 mph and the ambulance will average 50 mph.

In the questions that follow, round all approximate answers to two decimal places.



1. In the figure, x represents the distance downshore to the point where the ambulance is to meet the boat. Suppose the boat sails directly to shore. Here $x = 0$ miles.
   a. How far must the boat travel?
   b. How long will the boat take?
   c. How far must the ambulance travel to the hospital?
   d. How long will the ambulance take?
   e. How long will it take to get the passenger to the hospital?
2. Suppose that $x = 10$ miles.
   a. Determine the distance the boat must travel.
   b. How long will the boat take?
   c. How far must the ambulance travel to the hospital?
   d. How long will the ambulance take?
   e. How long will it take to get the passenger to the hospital?

3. Record your answers to the previous questions in the chart below and complete the chart by doing calculations similar to those you did in question 2.

x	Boat Distance	Boat time	Ambulance distance	Ambulance time	Total time
0					
10					
20					
30					
40					
50					
60					

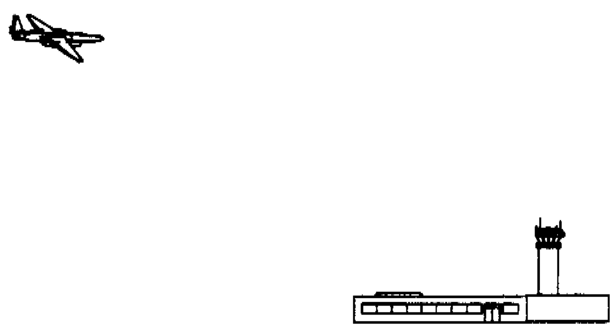
4. Which value of x results in the shortest time for the trip?

5. Try to shorten the total time required for the trip by using other values of x. Record the total time and the value of x for any better times you find. Compare your results with those of your classmates.

Source: Connecting Mathematics, Addenda Series, Grades 9-12. Reston: National Council of Teachers of Mathematics, 1991.

4. Indicators of Learning 2.2.2, 1.1.4, 2.4.2

A plane is 2 miles above the control tower. The air traffic controller sights the plane at an angle of elevation of 10°. What is the ground distance from the plane to the airport?



2.3 *Expectation: The student will formulate and justify conclusions.*

Indicators of Learning

2.3.1 The student will validate properties of geometric figures with appropriate technology and tools.

2.3.2 The student will identify and use inductive and deductive reasoning.

Comment

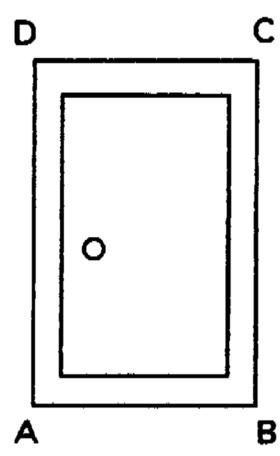
The following comment is specific to Expectation 2.3:

- Valid arguments may be in a variety of forms including: narrative, flowchart, or two-column proof.

Examples

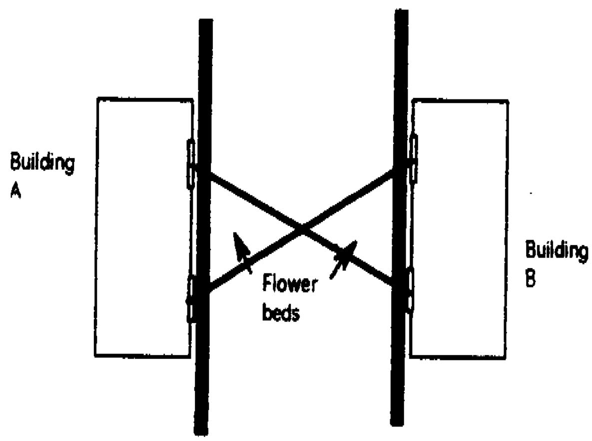
1. **Indicator 2.3.1**

A carpenter needs to square up the frame of a door. Using geometry, explain a process he might use. "Square up" is defined as forming right angles at each vertex.



2. **Indicator 2.3.2**

The roads in front of two buildings are parallel. The distance between the doors of each building is the same. Walkways connect opposite doors of each building forming two triangular flower beds as shown in the figure below. Use your knowledge of geometry to justify that the triangular regions containing the flower beds are the same size and shape.



3. **Indicators of Learning 2.3.1, 2.3.2**

Describe and carry out an investigation that uses inductive or deductive reasoning to find the sum of the exterior angles of a triangle. State the generalization, explain the geometry, and identify the type of reasoning used.

Note: This is an example that can be approached in many different ways: using interactive software, traditional tools (straightedge and protractor), or geometric properties. It is deemed appropriate that these various methods are acceptable and encouraged based on the type of course a student is in.

Source: A Core Curriculum, Addenda Series, Grades 9-12. Reston: National Council of Teachers of Mathematics, 1992. Used with permission.

2.4 <u>Expectation</u>: The student will apply concepts of measurement using tools and technology when appropriate.

<u>Indicators of Learning</u>

2.4.1 The student will measure and compute with appropriate precision.

2.4.2 The student will use algebraic and geometric properties to measure indirectly.

2.4.3 The student will estimate, calculate, and compare perimeter, circumference, area, volume, and surface area of two- and three-dimensional figures and their parts.

<u>Comment</u>

The following comment is specific to Expectation 2.4

- Students need to select the appropriate tool for the given measurement.

<u>Example</u>

1. **Indicator 2.4.1, 2.3.2**

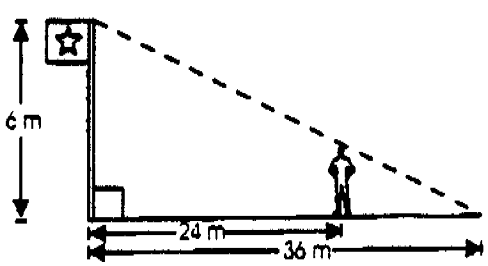
You are measuring the top of a large rectangular table to the nearest tenth of a meter, and find that it is 6.4 m by 2.1 m. Considering the given length and width, what would be the appropriate precision for the area?

a. 13.44 m^2 b. 13.4 m^2 c. 13 m^2

Justify your answer.

2. **Indicator 2.4.2, 2.3.2, 2.2.1, 1.1.4**

Juan is standing 24 m away from a 6 m flagpole which casts a 36 m shadow as shown.



How tall is Juan? Justify your answer.

3. Indicators of Learning 2.4.3, 2.1.2, 1.1.4, 2.3.2

You are asked to construct a rectangular container from a 9"x12" piece of sheet metal by removing a 2"x2" square from each corner and then folding the metal.

1. Draw a diagram to illustrate this problem.
2. Label the dimensions on your diagram.
3. Calculate the volume of the container that would be formed after you cut out the 2"x2" corners and fold the metal.
4. In order to reduce the amount of scrap metal that is left after you cut out the 2"x2" corners, it is suggested that you cut out corners that are only 1"x1". How would this affect the volume? Justify your conclusion.

GOAL 3: DATA ANALYSIS AND PROBABILITY

The student will demonstrate the ability to apply probability/statistical methods for representing and interpreting data and communicating results, using technology when needed.

3.1 Expectation: The student will collect, organize, and analyze statistical data.

Indicators of Learning

3.1.1 The student will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results.

3.1.2 The student will make predictions by constructing and using a line/curve of best fit.

3.1.3 The student will use the measures of central tendency and variability (mean, median, mode, range, interquartile range, quartile) to make informed decisions and predictions.

3.1.4 The student will recognize and communicate the use and misuse of statistics.

Comment

The following comment is specific to Expectation 3.1:

- Students may be asked to collect data in addition to having data provided.

Examples

1. Indicators of Learning 3.1.1, 3.1.2

These prices for used Ford Mustangs were taken from the *Milwaukee Journal* of January 28, **1990**.

Age	Price
8 yrs.	$ 1400
7 yrs.	2595
4 yrs.	4488
4 yrs.	5495
4 yrs.	4995
3 yrs.	6000
1 yr.	12895
11 yrs.	1350
10 yrs.	950
5 yrs.	1900
3 yrs.	5890
2 yrs.	6850

1. Make a scatter plot of age versus price. Put the age on the horizontal axis and price on the vertical axis.
2. Find the line of best fit. What is the equation of this line?
3. On the basis of your data and your equation, for how much would you have sold a six-year-old Mustang in 1990?
4. If you had $4000 to spend on a used Mustang, how old a car could you expect to buy?

Source: Data Analysis and Statistics Across the Curriculum, Addenda Series, Grades 9-12. Reston: National Council of Teachers of Mathematics, 1992. Used with permission.

2. **Indicators of Learning 3.1.3, 3.1.4, 2.3.2**

Three classes were in competition for an award for "best" scores on a mathematics test. Ten student scores were selected to represent each class's success. Which class would receive the award if you were the judge? Justify your answer.

Student	Class 1	Class 2	Class 3
1	200	603	410
2	400	603	410
3	530	603	480
4	545	603	556
5	640	603	603
6	680	603	603
7	725	603	650
8	750	603	726
9	770	603	796
10	790	603	796

3.2 Expectation: The student will apply the basic concepts of statistics or probability to real-world situations.

Indicators of Learning

3.2.1 The student will determine the number of ways an event can occur and compute the probability of such an event.

3.2.2 The student will use simulations or statistical inferences from data to estimate the probability of an event.

3.2.3 The student will make informed decisions and predictions based upon the result of simulations and data from research.

Comments

The following comments are specific to Expectation 3.2:

- Students are to select an appropriate simulation, conduct the simulation, and draw conclusions.
- Simulations could employ flipping a coin, tossing a die, or using a random number table.

Examples

1. **Indicator 3.2.1**

   Three pairs of a parent and a child have purchased tickets for six adjacent seats in a row for a baseball game.

   1. In how many ways can they be seated?
   2. In how many ways can they be seated if each pair is to sit together with the parent to the left of the child?
   3. In how many ways can they be seated if each pair is to sit together?
   4. In how many ways can they be seated if all parents are to sit together and all the children are to sit together?

2. **Indicators of Learning 3.2.2, 2.2.2, 2.3.2**

   What is the probability that three single-digit numbers will form the sides of a triangle? Simulate an experiment using the graphing calculator to generate random numbers. Justify your answer using geometry and probability.

3. Indicators of Learning 3.2.3, 3.1.2

Given the following data on "Oil Changes and Engine Repairs":

Oil changes per year	Cost of repair
3	$ 300
5	300
2	500
3	400
1	700
4	400
6	100
4	250
3	450
2	650
0	600
10	0
7	150

1. Make a scatter plot of oil changes versus cost of repair. Put the oil changes on the horizontal axis and the cost of repair on the vertical axis.
2. Find the line of best fit. What is the equation of this line?
3. Make a concluding statement from the data.

Source: Data Analysis and Statistics, Addenda Series, Grades 9-12. Reston: National Council of Teachers of Mathematics, 1992. Used with permission.

4. **Indicator 3.2.3**

The marketing division of a national candy company is having a promotional contest. A large 453-gram bag of candies will be awarded to the person who makes the best guess for the number of candies in the bag without opening or touching the bag. Design and conduct a process to win this contest. Explain the process you used and justify your conclusion.

5. **Indicators of Learning 3.2.3, 3.2.2**

A cereal company is placing three different toys in their new cereal. Each package will contain one toy. The toys are distributed in the following way: 20% of the packages contain toy A, 30% of the packages contain toy B, and 50% of the packages contain toy C. What is the least number of boxes of cereal will someone must buy in order to get all three prizes? Design and conduct a simulation to determine this. Explain the process you used and justify your conclusion.

MATCH OF MATHEMATICS CORE LEARNING GOALS AND SKILLS FOR SUCCESS CORE LEARNING GOALS

Shaded blocks indicate a positive match.

MATHEMATICS EXPECTATIONS	SKILLS FOR SUCCESS EXPECTATIONS																		
	1.1	1.2	1.3	1.4	1.5	2.1	2.2	2.3	2.4	3.1	3.2	3.3	3.4	4.1	4.2	4.3	5.1	5.2	5.3
1.1		■	■	■	■	■	■	■	■		■	■	■	■	■				
1.2		■	■	■	■	■	■	■	■	■	■	■		■	■				
2.1											■	■		■	■				
2.2		■	■		■	■		■	■			■	■	■	■				
2.3		■	■	■	■	■	■	■	■	■	■	■	■	■	■				
2.4		■		■	■		■	■	■	■	■			■	■				
3.1		■	■	■	■	■	■	■	■	■	■	■	■	■	■			■	■
3.2		■	■	■	■	■	■	■	■			■	■	■	■			■	■

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