

INTRODUCTION

The BG were developed collaboratively by Maryland secondary and college mathematics educators to define the mathematics needed to bridge the gap between the mathematics in the CLG and the mathematics needed to be successful in the first credit-bearing college mathematics course. The CLG were developed under the direction of the Maryland State Department of Education (MSDE) by Maryland mathematics educators to define the mathematics that all students should know and be able to do when they graduate from a Maryland public high school. The Maryland high school assessments have been developed to assess the CLG. At the college level the code of Maryland Regulations (COMAR) states that at each public institution in Maryland, the mathematics general education program “shall require at least ...one course in mathematics at or above the level of college algebra...”

In 1995 Maryland college mathematics professors reviewed the CLG. While they supported the CLG, the reviewers noted that they did not contain all the mathematics students need to be successful in the first credit-bearing course at the college level. When MSDE accepted the mathematics CLG, they asked the K-16 Council to oversee the development of the mathematics goals that bridge the gap between the mathematics in the CLG and the mathematics needed to be successful in the first credit-bearing college mathematics course. That process began in October 1996 as a collaborative effort between secondary and college mathematics educators. A final report from Bridge Goals Task Force I was accepted by the K-16 Council in June 1998.

Bridge Goals Task Force II was formed to further review the goals and to consider implementation issues. The second BG Task Force from 1999 – 2001 designed a research project to clarify the link between high school and college mathematics using the BG that were accepted in 1998. An assessment was developed and administered in two cycles to solicit performance data on the goals. The study included 5250 students - 2540 high school students and 2710 higher education students. The assessment focused on the linkages between three courses: high school’s Algebra 2 course; higher education’s first credit-bearing course, College Algebra; and higher education’s highest developmental course, Intermediate Algebra. The assessment produced data that provided a clearer picture of the content and rigor students need to be successful in the first credit-bearing college course. The data also gave evidence that high school’s Algebra 2 and higher Education’s Intermediate Algebra were similar in content and for the most part both contained the Bridge Goals. One of the results of this project was the development of assessment limits and skill statements for the BG. The Assumptions and Recommendations from the 1998 Bridge Goals Task Force I Report were also updated. In addition, minor revisions were made to the Expectations and Indicators. A second report was presented to the K-16 Workgroup in 2001.

Bridge Goals Task Force III extended the work of Task Force II by further testing the findings between Algebra 2, College Algebra, and Intermediate Algebra. Task Force III was able to narrow the focus of the BG review to the local level for a look at the

implications of implementation. Schools were selected by clusters, which included two high schools, a Community College and a 4-year institution within the same county or geographic region. This study included an additional 2850 students – 1020 high school students and 1830 higher education students. The BG were investigated within each cluster by:

- Analyzing student performance on the BG assessment for the three courses – Algebra 2, College Algebra, and Intermediate Algebra
- Analyzing curricular alignments between the courses within each cluster

As a result of the work of Task Force III, the BG have been slightly modified along with the assessment limits and skills statements. In addition, the assumptions have been modified from the 2001 document.

Bridge Goals Assumptions (Revised from 2001)

1. Competence in CLG is assumed. The BG do not revisit the CLG, and the BG cannot be assumed to function outside the context of the CLG. In some cases, the BG build on the CLG, emphasizing an extension of mathematical concepts, or further depth (for example, graphical understanding for a wider class of functions). In other cases, the BG do not mention content areas of the CLG. For example, the BG do not contain any specifics concerning data analysis or geometry. This is not to say that data analysis and geometry are not important, nor that data analysis and geometry are not part of the expectations of a college general education course, but rather that statistics and geometry are adequately covered in the CLG.
2. The Bridge Goals have been developed on the foundation of the Core Learning Goals and the competencies necessary for entrance into the first credit-bearing college-level general education mathematics courses. General Education mathematics courses are identified as having the same pre-requisite as College Algebra. (COMAR regulations) The BG define the minimal criteria needed across the state of Maryland to be successful in the first credit-bearing mathematics course at the college level. The Bridge Goals should be viewed as the “floor”, not the “ceiling”, of mathematics of a college bound high school graduate.
3. Periodic analysis of student performance in the three courses: Algebra 2, Intermediate Algebra, and College Algebra is important to maintain the consistent linkages between the three.
4. The BG would be used as a guide for developing high school curriculum for mathematics courses beyond the CLG for students going to college.
5. The Bridge Goals would be separate from Maryland’s school accountability. The Task Force assumes the BG are uncoupled from any mechanism for evaluating schools.
6. The Bridge Goals and CLG assume appropriate technology is available to all students. In light of the CLG, it is assumed each student will have available (either by individual purchase or provided by the school district) a graphing calculator.

The use of technology varies across and among institutions of higher education. The task force recognizes that technology is perhaps the one area where events will overtake policy. The term, “appropriate technology” is a moving target; we cannot predict what technology will be available even half a decade from now. However, although not all expectations in the Bridge Goals explicitly mention technology, the assumption is that all students have access to, and working familiarity with, appropriate technology.

7. Because the Task Force does not report, directly or indirectly, to any higher education policy-making body, there was much discussion concerning what implications the Bridge Goals will have to admission in college credit-bearing courses. This was often phrased in the context that students would not be inclined to take any course or assessment based on the Bridge Goals unless there was a tangible outcome for them. Maryland colleges and universities include a variety of instruments in determining placement of students, and it is assumed that an assessment based on the Bridge Goals will become one of the instruments
8. The Task Force assumes that there will be sufficient resources available to implement the Bridge Goals. Three major areas requiring resources are: providing appropriate technology for all students; professional development for pre-service and in-service teachers; and the continued investigation into a BG assessment instrument.

GOAL 1: INTEGRATION INTO BROADER KNOWLEDGE

The student will develop, analyze, communicate, and apply models to real-world situations, using the language of mathematics and appropriate technology.

1.1 ***Expectation:** The student will model and interpret real-world situations, using the language of mathematics and appropriate technology.*

Indicators

1.1.1 the student will determine and interpret a linear function when given a graph, table of values, essential characteristics of the function, or a verbal description of a real-world situation.

Assessment Limits:

- The majority of these items should be in context.
- Absolute value functions are in one variable.
- Essential characteristics are any points on the line, x- and *y-intercepts, *slope.
- *Students should be able to perform these skills with and without the use of a graphing calculator.

Skill Statement:

Given one or more of the following:

- a written description
- a graph
- a *table of values
- an *equation
- two or more essential characteristics
- an absolute value equation

the student will be able to do each of the following:

- write and/or solve an equation or an inequality that models the situation
- graph the function
- find and/or interpret the meaning of any essential characteristics in the context of the problem.

1.1.2 The student will determine and interpret a quadratic function when given a graph, table of values, essential characteristics of the function, or a verbal description of a real-world situation.

Assessment Limits:

- The majority of the items should be in context.
- Essential characteristics are zeros, vertex (maximum or minimum), y-intercept, increasing and decreasing behavior.
- A table of values must include rational zeros and at least one other point.
- All have real zeros.

Skill Statement:

Given one or more of the following:

- a written description
- a graph
- a table of values
- a function in equation form

the student will be able to do each of the following:

- find one or more of the essential characteristics
- write the function in equation form
- graph the function
- predict the value of $f(x)$ for a given number x
- find x for a given value of $f(x)$.

1.1.3 The student will determine and interpret an exponential function when given a graph, table of values, essential characteristics of the function, or a verbal description of a real-world situation.

Assessment Limits:

- The majority of these items are to be in context.
- Essential characteristics are y-intercepts, asymptotes, increasing or decreasing.
- For $f(x) = a b^x$, $b > 0$, a and b are rational numbers, b is not 1.
- The y-values for $x = 0$ and $x = 1$ will be given.

Skill Statement:

Given one or more of the following

- a written description
- a table of values
- a graph
- a function in equation form

the student will be able to do each of the following:

- find one or more of the essential characteristics
- write the function in equation form
- graph the function
- predict the value of $f(x)$ for a given number x
- find x for a given value of $f(x)$.

1.2 Expectation: Given an appropriate real-world situation, the student will choose an appropriate linear, quadratic, or exponential model and apply that model to solve the problem.

Assessment Limits:

- The majority of these items include a written description of a real-world situation.

Skill Statement:

- Given a scatter plot of approximately linear data, the student will write an equation of best fit and/or use that equation to find values for x or $f(x)$ using a graphing calculator.
- Given a written description and/or a table of values of a function, the students will recognize that the function is linear, quadratic, or exponential and/or write the appropriate equation that models the.

1.3 Expectation: *The student will communicate the mathematical results in a meaningful manner.*

Indicators

1.3.1 The student will describe the reasoning and processes used in order to reach the solution to a problem.

Assessment Limits

- This indicator is assessed through the implementation of the Core Learning Goal rubric for the free response items. (See attached)

1.3.2 The student will ascribe a meaning to the solution in the context of the problem and consider the reasonableness of the solution.

Assessment Limits

- This indicator is assessed through the implementation of the Core Learning Goal rubric for the free response items. (See attached)

GOAL 2: MATHEMATICAL CONCEPTS, LANGUAGE, AND SKILLS

The student will demonstrate the ability to analyze a wide variety of patterns and functional relationships using the language of mathematics and appropriate technology.

2.1 Expectation: *The student will be familiar with basic terminology and notation of functions.*

Indicators

2.1.1 The student will identify and use alternative representations of functions.

Assessment Limits:

- These items are not in context.
- Absolute value functions are in one variable.

Skill Statements:

Given one or more of the following:

- a table of values
- a graph
- an equation
- a verbal description

the student will be able to do each of the following:

- find a value for x or $f(x)$
- find real roots
- find maximum and/or minimum
- find intervals on which the function is increasing and/or decreasing.

Given an absolute value function, the student will graph the function and/or calculate numeric value of the function.

2.1.2 The student will identify the domain, range, or rule of a function.

Assessment Limits:

- Vertical and horizontal lines are included.
- Functions with restricted domain and/or range are included.
- Absolute value functions are in one or two variables.
- Rational functions should have denominators that are linear/quadratic/sum and/or difference of two cubes in factored form.

Skill Statements:

Given one or more of the following:

- a graph of a linear or non-linear function or relation
- an equation over a specific interval
- a written description of a real-world situation with a restricted domain
- a simple rational function

the student will be able to do each of the following

- describe the domain
- describe the range.

Given the equation of a function, the student will produce the graph and describe the domain and range using inequalities.

2.2 Expectation: *The student will perform a variety of operations and geometrical transformations on functions.*

Indicators

2.2.1 The student will add, subtract, multiply, and divide functions.

Assessment Limits:

- Items involving factoring will be restricted to quadratics or the sum or difference of two cubes.
- Long division is restricted to linear binomial or monomial terms in the denominator.

2.2.2 The student will find the composition of two functions and determine algebraically and/or graphically if two functions are inverses.

Assessment Limits:

- Functions given in equation form can include linear, quadratic, exponential or forms such as $f(x) = (ax+b)/(cx+d)$.

Skill Statements:

- Given a function in equation form, the student will find the inverse function in equation form.
- Given a one-to-one function as a graph, the student will graph the inverse of the function.
- Given a function as a table of values, the student will determine the domain and/or range of the inverse of the function.

2.2.3 The student will perform translations, reflections, and dilations on functions.

Assessment Limits:

- Translations are either vertical or horizontal shifts.
- Dilations either shrink or stretch a function.
- This indicator assesses recognition of translations, reflections, and dilations on functions.
- Transformations for absolute value function in one or two variables are restricted to translations and reflections. They do not include dilations.
- Exponential functions are restricted to translations.

Skill Statements:

- Given a verbal description of a transformed linear, quadratic, or exponential function, the student will write the function in equation form.
- A transformed linear, quadratic, or exponential function in equation form, the student will give a verbal description of the transformation.

2.3 Expectation: *The student will identify linear and nonlinear functions expressed numerically, algebraically, and graphically.*

Assessment Limits:

- The items may have no context given.
- Graphs may include piece-wise functions.

Skill Statements:

Given one or more of the following:

- a table of values
- a graph

the student will be able to do each of the following:

- choose the correct equation or graph from the same family of functions
- choose the correct equation or graph from a variety of families of functions.

2.4 Expectation: *The student will describe or graph notable features of a function using standard mathematical terminology and appropriate technology.*

Assessment Limits:

- Essential characteristics of a linear, quadratic, or exponential function are those listed for 1.1.1, 1.1.2, and 1.1.3.
- Transformations for absolute value function in one or two variables are restricted to translations and reflections. They do not include dilations.

Skill Statements:

- Given one or more of the essential characteristics of a function, the student will graph the function.
- Given the equation form of a linear, quadratic, or exponential function, the student will find one or more required essential characteristic and/or graph the function.

2.5 Expectation: The student will use numerical, algebraic, and graphical representations of functions in order to solve equations and inequalities.

Assessment Limits:

- Absolute value equations and inequalities are in one variable.
- Radical equations that lead to a quadratic are restricted to square roots.
- Rational equations will lead to a linear or quadratic equation.
- Simple rational inequalities will lead to a linear inequality.
- Exponential equations are either of the form $f(x) = a b^x$, $b > 0$, a and b are rational numbers, b is not 1 or the form $c^{nx+d} = g^{mx+f}$, where c and g are powers of the same base.

Skill Statements:

- Given an equation or inequality expressed numerically, algebraically, or graphically, the student will find the solution. For free-response items the student will also justify their method and/or solution.
- Given a quadratic inequality, the student will find the solution. For free-response items the student will also justify their method and/or solution.

2.6 Expectation: The student will solve algebraically two-variable systems of linear equations and solve graphically two-variable systems of linear inequalities.

2.7 Expectation: The student will use the appropriate skills to assist in the analysis of functions.

Indicators

2.7.1 The student will add, subtract, multiply, and divide simple rational expressions.

Assessment Limits:

- Items may include monomial, quadratics, and the sum and difference of two cubes.

2.7.2 The student will solve quadratic equations of the form $y = ax^2 + bx + c$ by factoring and the quadratic formula.

Assessment Limits:

- The coefficients (a), (b) and constant(c) are integers and when factoring, $a = 1$.
- The solutions are real numbers.

2.7.3 The student will operate with rational exponents.

2.7.4 The student will add, subtract, multiply, and divide radicals in both radical and exponent form.

Assessment Limits:

- Rationalizing denominators are restricted to square roots.
- Radicals containing a numerical coefficient are restricted to square roots and cube roots.

2.8 Expectation *The student will solve literal equations and formulas.*

Assessment Limits:

- Problems may include addition/subtraction and multiplication/division properties of equality, factoring a common factor, and terms that are rational.

HSA Mathematics Rubric for Brief Constructed Response Items

- 3 The response indicates **application** of a reasonable strategy that leads to a correct solution in the context of the problem. The **representations** are essentially correct. The **explanation** and/or **justification** is logically sound, clearly presented, fully developed, supports the solution, and does not contain significant mathematical errors. The response demonstrates a complete understanding and **analysis** of the problem.
- 2 The response indicates **application** of a reasonable strategy that may be incomplete or undeveloped. It may or may not lead to a correct solution. The **representations** are fundamentally correct. The **explanation** and/or **justification** supports the solution and is plausible, although it may not be well developed or complete. The response demonstrates a conceptual understanding and **analysis** of the problem.
- 1 The response indicates little or no attempt to **apply** a reasonable strategy or applies an inappropriate strategy. It may or may not have the correct answer. The **representations** are incomplete or missing. The **explanation** and/or **justification** reveals serious flaws in reasoning. The explanation and/or justification may be incomplete or missing. The response demonstrates a minimal understanding and **analysis** of the problem.
- 0 The response is completely incorrect or irrelevant. There may be no response, or the response may state, "I don't know."

Explanation refers to the student using the language of mathematics to communicate how the student arrived at the solution.

Justification refers to the student using mathematical principles to support the reasoning used to solve the problem or to demonstrate that the solution is correct. This could include the appropriate definitions, postulates and theorems.

Essentially correct representations may contain a few minor errors such as missing labels, reversed axes, or scales that are not uniform.

Fundamentally correct representations may contain several minor errors such as missing labels, reversed axes, or scales that are not uniform.

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Project Sites for Bridge Goals Assessment

Community Colleges

Allegany College of Maryland
Anne Arundel
Community College of Baltimore – Catonsville
Cecil
Chesapeake
Frederick
Garrett College
Hagerstown
Howard
Montgomery College
Prince George's

Four-Year Colleges

Frostburg
Hood
Salisbury
University of Maryland Baltimore County
University of Maryland College Park

County High Schools

Allegany
Anne Arundel
Baltimore City
Baltimore
Cecil
Dorchester
Frederick
Garrett
Howard
Kent
Montgomery
Prince George's
Queen Anne's
Talbot
Washington
Wicomico

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