COMMENTARY FOR THE **AAT MATHEMATICS GROUP REPORT**, February 2004:

1. There is agreement by the AAT Mathematics Group that the AAT mathematics document include the following:

1. 18 Outcomes, which are directly related to the basic topics comprising the 3-course sequence Calculus I, II, and III, as well as lower-division Linear Algebra.

2. Either of the following two tracks for the science component:

A. Track 1: 2-course Calculus-based Physics I-II
B. Track 2: 2-course Algebra-based Physics I-II or 2-course Chemistry I-II.

Among the 4-year institutions the assignment of tracks are as follows:

Bowie State University: Track 1
Coppin State College: Track 1
Frostburg State University: Track 1 or Track 2
Hood College: Track 1 or Track 2
Morgan State University: Track 1
College of Notre Dame: Track 1 or Track 2
Mt. St. Mary's College: Track 1 or Track 2
Salisbury University: Track 1 or Track 2
St. Mary's College: Track 1 or Track 2
Towson University: Track 1
UMBC: Track 1
UMCP: Track 1 or Track 2
UMES: Track 1
Washington College: Track 1 or Track 2

Note: As for Linear Algebra, this lower-division course should transfer as such to any public 4-year institution statewide, irrespective of the numbering at the 4-year institution.

Respectfully submitted,
Dan Symancyk and Denny Gulick, co-chair

Standard: Linear Algebra, Calculus

Teacher candidates will be able to:

Outcomes	**Indicators**	**Assessment Type**	**Sample Assessment Tasks**
1. Understand matrices and their applications	a. Perform matrix operations (addition, subtraction, multiplication, scalar multiplication, and transposition); demonstrate knowledge of properties of the above operations. b. Solve linear systems using elementary row operations. c. Have a working knowledge of inverse matrices (existence of, calculation of, and use in solving systems of equations) d. Have a working knowledge of determinants (existence of, operations on, calculation of, and properties of) and relationship to geometry.	• Extended response	• Given a square matrix, use elementary row operations to determine whether the inverse exists, and if so, find the inverse.
2. Understand vectors and vector spaces and their applications.	a. Recognize vector spaces and subspaces of vector spaces. b. Operate on vectors in R^n and describe the geometrical interpretations of those operations. c. Identify when two or more vectors are linearly independent and/or span. d. Recognize a basis for a vector space, and determine the dimension of a vector space.	• Extended response	• Given a subset of a vector space, determine whether the subset is a subspace.

	e. Identify orthogonal vectors. f. Represent vectors with respect to different bases.		
3. Understand linear transformations and their applications	a. Recognize linear transformations and their connection to matrices. b. Determine null space, image, and rank of a linear transformation. c. Determine whether two vector spaces are isomorphic. d. Find and analyze eigenvalues, eigenvectors, characteristic polynomials, and eigenspaces. e. Perform the diagonalization process on a square matrix.	• Extended response	• Given the vector space of polynomials of degree less than or equal to four, determine the matrix of the linear transformation given by differentiation with respect to the standard basis.
4. Use technology to assist with calculations and explorations	a. Having demonstrated manual computational skills, use technology to assist with those computations where appropriate (e.g., operations on matrices, calculating determinants). b. Explore use of technology in complex "real world" computations (e.g., solving linear systems of equations, finding L-U factorization of matrices, and finding eigenvalues of matrices). c. Use technology to expand explorations of linear algebra.	• Projects • Extended response	• Use technology to explore the images of the unit square in the first quadrant under several linear transformations.
5. Identify the properties of basic classes of functions. (Here	a. Verify properties of symmetric, inverse, and composite functions for a collection of algebraic, exponential,	• Short answers, extended responses, graphs,	• Given the graph of a function, tell whether it is of exponential,

"functions" are algebraic, inverse, exponential [including hyperbolic], logarithmic, trigonometric.)	logarithmic, and trigonometric functions.	quizzes, tests	logarithmic, polynomial, or trigonometric type.
6. Calculate the limits of functions.	a. Analyze problems using the Squeezing Theorem, one-sided limits, infinite limits, l'Hôpital's Rule.	• Short answers, extended responses, graphs, quizzes and tests.	• Evaluate a limit using l'Hôpital's Rule
7. Analyze continuity of a function	a. Identify continuity and piecewise continuity of functions and analyze properties of continuity through the Intermediate Value Theorem.	• Short answers, extended responses, graphs, quizzes and tests	• Use the Intermediate Value Theorem to show that the range of the sine function contains all numbers in the interval [-1, 1].
8. Find the derivatives of functions numerically, algebraically, and graphically.	a. Calculate the derivative of a function (using basic rules of differentiation, including the chain rule and implicit differentiation) and use it to find the slope, tangent, higher derivatives. b. Estimate approximate values of functions (with technology), and find the relation between the derivative of a function and its inverse.	• Short answers, extended responses, graphs, quizzes and tests	• Determine the values (if any) where the line tangent to a given third degree polynomial is horizontal.
9. Apply the derivative to diverse situations.	a. Apply the derivative to find related rates, velocity and acceleration from position, properties of graphs of functions (including relative extrema, asymptotes, concavity), solutions of maximum and minimum problems, and exponential growth and decay. b. Explain the uses of Rolle's Theorem and the Mean Value Theorem.	• Short answers, extended responses, graphs, quizzes and tests	• Find the maximum volume of a right circular cylinder that is inscribed in a given sphere.
10. Calculate definite integrals and find	a. Apply Riemann Sums, the Fundamental Theorem of Calculus,	• Short answers, extended responses, graphs	• Calculate the area of the region bounded above by the sine

indefinite integrals.	algebraic and trigonometric substitutions, integration by parts, and partial fractions to find integrals. Estimate values of integrals by means of Simpson's Rule (with technology). b. Explain why differentiation and integration are inverse processes, and indicate the historical roles of Newton and Leibniz for the calculus.	quizzes, tests short essays	function, below by the x-axis, between $x = 0$ and $x = \pi$.
11. Solve applied problems related to integration.	a. Using integration, find solutions to problems involving area, volume, surface area, work, moments, and length of a curve, as well as position and velocity from known acceleration.	• Short answers, extended responses, graphs, quizzes and tests	• Find the area of the region formed by a given ellipse.
12. Analyze the convergence or divergence of sequences and series.	a. Use convergence properties of sequences to determine the convergence or divergence of a given sequence. b. Use the convergence tests (nth term test, integral test, ratio test, alternate series test) to determine the convergence or divergence of given series. c. Find the power series and Taylor series for given functions with the Lagrange Remainder Formula. d. Apply Taylor's Theorem, absolute convergence to power series, and find the radius of convergence of a power series.	• Short answers, extended responses, graphs, quizzes and tests	• Find the Taylor series for the sine function, and determine the radius of convergence of the Taylor series.
13. Graph and analyze polar equations,	a. Analyze functions given in polar form or in parametric form.	• Short answers, extended responses, graphs,	• Discuss the properties of the cycloid.

parametric equations, and conic sections.	b. Analyze rectangular forms of conic sections. c. Calculate lengths and areas related to polar and parametric functions.	quizzes and tests	
14. Solve elementary differential equations.	a. Explain basic definitions relative to differential equations and solve separable differential equations. b. Find approximate solutions, for example, using Euler's method. c. Sketch a solution given a slope-field.	• Short answers, extended responses, graphs, quizzes and tests	• Solve the differential equations for the exponential growth and decay.
15. Explain properties of vectors and vector-valued functions.	a. Calculate dot products, cross products, distances between points and lines in space. b. Find derivatives, tangents, normals, curvature for parameterized curves (using technology).	• Short answers, extended responses, graphs, quizzes and tests	• Find the distance between a given point and a given line in space.
16. Apply differentiation rules to various multivariable functions. Identify these properties of quadric surfaces.	a. Find directional derivatives, gradients, tangent planes, and approximations (using technology) by means of partial derivatives. b. Find extreme values of multivariable functions, including the use of Lagrange multipliers. c. Describe geometric properties of multivariable functions, including level curves and quadric surfaces.	• Short answers, extended responses, graphs, quizzes and tests	• Find the plane tangent to the graph of a given paraboloid.
17. Evaluate multiple integrals.	a. Evaluate double and triple integrals using rectangular, cylindrical and spherical coordinates, as well as change of variables. b. Find volumes, mass and moments of objects in space.	• Short answers, extended responses, graphs, quizzes and tests	• Find the volume of the solid region that lies inside a given cone and given sphere
18. Explain properties of	a. Explain and calculate the divergence,	• Short answers, extended	• Show that a given line integral is

vector fields and evaluate various vector field derivatives and integrals.	gradient and curl of a given function. b. Evaluate line integrals, and surface integrals by means of the Fundamental Theorem of Line Integrals, Green's Theorem, Stokes's Theorem and the Divergence Theorem.	responses, graphs, quizzes and tests	independent of path.

The above outcomes are included in the following courses:

Calculus I , II, and III (for science, engineering, and mathematics majors)
Linear Algebra (sophomore level)

To meet their general education science requirements, mathematics education majors must complete Calculus-based Physics I and II for Track I, or either Algebra-based Physics I and II or Chemistry I and II for Track II.