rational numbers: Numbers that can be expressed as an integer, as a quotient of integers (such as $\frac{1}{2}$,

 $\frac{4}{3}$, 7), or as a decimal where the decimal part is either finite or repeats infinitely (such as 2.75 and 33.3333...) are considered rational numbers.

- *irrational numbers:* A number is irrational because its value cannot be written as either a finite or a repeating decimal such as π and $\sqrt{2}$.
- *real number system:* The set of numbers consisting of rational and irrational numbers make up the real number system.
- *truncate:* In this estimation strategy, a number is shortened by dropping one or more digits after the decimal point (i.e., 234.56 is truncated to the tenth's place → 234.5 by dropping the digit 6 in the hundredth's place).
- *properties of integer exponents:* These properties include product of powers, quotient of powers, negative exponents, zero exponent, and power of powers.

product of powers: Add the exponents $x^5 \cdot x^3 = x^8$ OR $2^5 \cdot 2^3 = 2^8$ *quotient of powers:* Subtract the exponents $\frac{x^5}{x^3} = x^2$ OR $x^5 \div x^3 = x^2$ OR $\frac{2^5}{2^3} = 2^2$ *negative exponents:* $x^{-3} = \frac{1}{x \cdot x \cdot x} = \frac{1}{x^3}$ OR $2^{-3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8}$ *zero exponents:* Using quotient of powers where exponents are subtracted $\frac{x^5}{x^5} = x^0$, and $\frac{x^5}{x^5} = 1$, therefore $x^0 = 1$ OR $2^0 = 1$ *power of powers:* Multiply the exponents $(x^4)^2 = (x \cdot x \cdot x \cdot x)^2 = (x \cdot x \cdot x \cdot x) (x \cdot x \cdot x \cdot x)$. Therefore $(x^4)^2 = x^8$ OR $(2^4)^2 = (2 \cdot 2 \cdot 2 \cdot 2)^2 = (2 \cdot 2 \cdot 2 \cdot 2) (2 \cdot 2 \cdot 2 \cdot 2)$. Therefore $(2^4)^2 = 2^8$

perfect square: A perfect square is the product of a number multiplied by itself.

Examples: $4 \cdot 4 = 16$, therefore 16 is a perfect square of 4;

 $-6 \cdot -6 = 36$, therefore 36 is a perfect square of -6;

$$\frac{7}{10} \bullet \frac{7}{10} = \frac{49}{100}$$
, therefore $\frac{49}{100}$ is a perfect square of $\frac{7}{10}$

perfect cubes: A perfect cube is the product of a number multiplied by itself twice.

Examples: $2 \cdot 2 \cdot 2 = 8$, therefore 8 is a perfect cube of 2

 $5 \cdot 5 \cdot 5 = 125$, therefore 125 is a perfect cube of $5 \cdot 5 = 125$, therefore $125 \cdot 5 = 125$.

 $\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64}$, therefore $\frac{27}{64}$ is a perfect cube of $\frac{3}{4}$

square roots ($\sqrt{}$) A square root of a number is a value which, when used as a factor two times produces the given number.

Example: $\sqrt{144}$ (read as square root of 144) is 12 because

 $12 \bullet 12 = 12^2 = 144.$

cube roots $(\sqrt[3]{})$: A cube root of a number is a value which, when used as a factor three times produces the given number.

Example: $\sqrt[3]{216}$ (read as cube root of 216) is 6 because 6 • 6 • 6 = 6³ = 216

- *principal (positive) roots and negative roots:* A positive number has two square roots. The principal root is positive and the other root is negative. For example the square roots of 121 are 11 and $^{-11}$ because $11^2 = 121$ and $(^{-11})^2 = 121$.
- *integer powers of 10:* Integer powers of 10 are numbers with a base of 10 and an exponent that is an integer.

Examples of positive: $10^1 = 10$; $10^2 = 100$; $10^3 = 1,000$; $10^4 = 10,000$; $10^5 = 100,000$ Examples of negative: $10^{-1} = \frac{1}{10}$; $10^{-2} = \frac{1}{100}$; $10^{-3} = \frac{1}{1,000}$; $10^{-4} = \frac{1}{10,000}$; $10^{-5} = \frac{1}{100,000}$ Example of zero: $10^0 = 1$

scientific notation: A number in scientific notation is written as the product of two factors. The first factor is a number greater than or equal to 1 and less than 10; the second factor is an integer power of 10.

Example: 37,482,000 is written 3.7482 x 10⁷ 0.00000037482 is written 3.7482 x 10⁷

decimal notation: Notation refers to symbols that denote quantities and operations. Values written in decimal notation use a decimal point to differentiate between whole number values and mixed number values, for example 132 versus 132.5. Mixed number values also can be written in fraction notation, for example $132\frac{1}{2}$.

slope:



Slope of a line is a ratio that describes the steepness of a line. Slope is usually written in fraction form that places the value of the horizontal change along the x-axis in the denominator of the fraction and the vertical change along the y-axis in the numerator. In the diagram, the horizontal change (run) is 2, while the vertical change (rise) is

3. Thus, the slope of the line is $\frac{3}{2}$.

$$slope = \frac{vertical \ change}{horizontal \ change} = \frac{rise}{run}$$

- *unit rate:* Unit rate is the ratio of two different measurements in which the second term is 1. Example: 6:1, 6 miles/one gallon, $\frac{6 \text{ miles}}{1 \text{ gallon}}$ (In fraction form, the denominator is 1.)
- *proportional relationship:* A comparison of two variable quantities having a fixed (constant) ratio is considered to be a proportional relationship. Example: 50 miles on 3 gallons is a proportional relationship to 100 miles on 6 gallons and 150 miles on 9 gallons.

constant rate of change/slope:



The line graphed on this coordinate plane has a slope of $\frac{3}{2}$. This means as one moves up or down the line, the ratio of the change in units on the x-axis, 2, is constant with the change in units on the y-axis, 3.

slope =
$$\frac{vertical change}{horizontal change} = \frac{rise}{run}$$

direct variation:



When two variables are related in such a way that the ratio of their values always remains the same, the two variables are said to be in direct variation. The line of this ratio always intersects the origin.

In the diagram shown, the vertical change for the line, 500 (grams), is in direct variation with its horizontal change, 100 (E). In other words, the vertical change is in direct variation to the horizontal change in a ratio of $\frac{5}{1}$.



```
zero slope:
```



undefined slope:

As shown in the diagram, when the slope of a line is undefined, the line is vertical and parallels the *y*-axis. As one moves up or down a line, the value for *x* does not change. The *x*-value for the line on the right is always 3, regardless of the *y*-value.

We know that division by zero is impossible because regardless of the value of the dividend, it can never be reproduced through multiplication by a divisor of zero because $0 \times 0 = 0$.

So, if we think of $\frac{y}{x}$ as $\frac{all \ values}{0}$, then the slope of a vertical line is incapable of being defined. For example, if you move on the line from the point (3, 7) to the point (3, 2), then $\frac{y}{x} = \frac{-5}{0} =$ undefined. Conversely, if you move from (3, 2) to (3, 7), then $\frac{y}{x} = \frac{5}{0} =$ undefined.

As shown in the diagram, when the slope of a line is zero, the line is horizontal and parallels the *x*-axis. As one moves left or right on the line, the value for y does not change. In the diagram, the y-value is always 5, regardless of the *x*-value.

If we think of $\frac{y}{x}$ as $\frac{0}{all \, values}$, then zero divided by any value is always zero.

For example, if you move on the line from the point (⁻4, 5) to the point (3, 5), then $\frac{y}{x} = \frac{0}{7} = 0$. Conversely, if you move from (3, 5) to (⁻4, 5), then $\frac{y}{x} = \frac{0}{-7} = 0$.



one solution: An example of a linear equation in one variable with one solution is x = 5.

infinitely many solutions: An example of a linear equation in one variable with infinitely many solutions is x = x.

no solution: An example of a linear equation in one variable with no solutions is x + 1 = x + 2.

families of graphs: The graph below shows a few family members for the linear equation y = x.

from 2 from 2 from 2

The red line, y = x, is considered the "parent." All members of a family of graphs have the same slope, but different y-intercepts. Families of graphs can have infinitely many members.

equivalent equation: Equations with the same value are equivalent, for example: 2 + 3 = 11 - 6 is equivalent to 5 = 5 OR y +3 = 5 is equivalent to y = 2.

distributive property: The distributive property states that for all numbers a, b, and c, a(b+c) = ab + ac

Examples are
$$a(b + c) = ab + ac$$
 and $3x(x+4) = 3x(x) + 3x(4)$
= $3x^2+12x$

collecting like terms: This process is also known as "combining" like terms. Example: 2x + 3 + 5x = 7x + 3

Solve Systems of Equations:

1. Elimination by addition or subtraction:

2x + y = 9 $3x - y = 16$	Using elimination by addit will cancel out, and $x = 5$.	ion or subtraction	, the y	variables	$\frac{2x+}{3x-}$ 5x	y = 9 $y = 16$ $= 25$ $= 5$
Once <i>y</i> has be determine the substituted into to determine the	en eliminated to value of <i>x</i> , 5 can be o either of the equations ne value of <i>y</i> .	2(5) + y = 910 + y = 9y = -1	or	3(5) - y = 1 15 - y = 1 -y = 1 y = -y = 1	x 16 16 1 -1	_ 3

The solution to the system of equations is: (x, y) = (5, -1)

2. elimination by substitution :

2x + y = 93x - y = 16

Solve for x or y in either equation. In this example it is easier to solve for y because it has a coefficient of 1.

2x + y = 9Therefore: 3x - (9 - 2x) = 16 (substituted 9-2x for y) 3x - 9 + 2x = 16 5x - 9 = 16 + 9 = + 9 5x = 25 x = 5

Since x = 5, substitute 5 for x in either equation and then solve for y. 2x + y = 9 becomes 2(5) + y = 9 10 + y = 9y = -1

The solution to the system of equations is: (x, y) = (5, -1)

3. graphically: Graph each equation. Where they intersect on the coordinate plane is the solution.

2x + y = 93x - y = 16

function: A function is an association of exactly one value or object from one set of values or objects (the range) with each value or object from another set (the domain). The equation y = 3x + 5 defines y as a function of x, with the domain (x values) specified as the set of all real numbers. Thus, when y is a function of x, a value of y (the range) is associated with each real-number value of x by multiplying it by 3 and adding 5.

Example: y = 3x + 5Domain: set of all x values Range : set of all y values output dependent

- *graph of a function:* The graph of a function is the set of ordered pairs (x, y) for each input (domain) value (x) and its corresponding output (range) value (y). For the equation y = 3x + 5, ordered pairs such as ($^{-}5$, $^{-}10$), ($^{-}2$, $^{-}1$), (0, 5), (3, 14), and (300, 305), are among an infinite number of ordered pairs.
- *function notation:* The expression "f(x)" means "plug a value for x into a formula f"; the expression does *not* mean "multiply f and x." In function notation, the "x" in "f(x)" is called "the argument of the function", or just "the argument". So if we are given "f(2)" the "argument" is "2".

We evaluate "f(x)" just as we would evaluate "y".

• Given $f(x) = x^2 + 2x - 1$, find f(2)

$$f(2) = (2)^{2} + 2(2) - 1$$

= 4 + 4 - 1

• Given $f(x) = x^2 + 2x - 1$, find f(-3)

$$f(-3) = (-3)^2 + 2(-3) - 1$$

= 9 - 6 - 1
= 2

functional relationship: A functional relationship describes an association of exactly one value or object from one set of values or objects (the range) with each value or object from another set (the domain).

- *constant rate of change/slope:* Anything that goes up by *x* number of units for each *y* value every time is a constant rate of change. A constant rate of change increases or decreases by the same amount for every trial. The slope of a line has a constant rate of change.
- *y-intercept:* The point at which a line crosses the y-axis (or the y-coordinate of that point) is a y-intercept.
- *linear function:* A linear function is a first-degree polynomial function of one variable. This type of function is known as "linear" because it includes the functions whose graph on the coordinate plane is a straight line. A linear function can be written as: $x \rightarrow ax + b$ or f(x) = mx + b
- *non-linear function:* Equations whose graphs are not straight lines are called nonlinear functions. Some nonlinear functions have specific names. A "quadratic function" is nonlinear and has an equation in the form of $y = ax^2 + bx + c$, where $a \neq 0$. Another nonlinear function is a "cubic function". A cubic function has an equation in the form of $y = ax^3 + bx^2 + cx + d$, where $a \neq 0$.
- *interval of a function:* A set of numbers containing all numbers between two given numbers (the end points) and one, both or neither end point.
- *transformations:* anytime you move, shrink, or enlarge a figure, you have to make a transformation of that figure. This kind of transformation includes rotations, reflections, translations and dilations.





- *rotation:* A rotation is also called a turn. Rotating a figure means "turning" the figure around a point. The point can be on the figure or it can be some other point. The shape still has the same size, area, angles and line lengths.
- *reflections:* A reflection is also called a flip. This transformation occurs when a figure is "flipped" over a line. Each point in a reflection image is the same distance from the line as the corresponding point in the original shape. The shape still has the same size, area, angles and line lengths.
- *translations:* A reflection is also called a slide. In a translation, every point in the figure "slides" the same distance in the same direction. The shape still has the same size, area, angles and line lengths.
- *dilations:* Dilation is also called resizing. The shape becomes bigger or smaller. If you have to dilate or "resize" to make one shape become another, they are similar. These figures are *non-rigid* because they do not stay the same.
- *are taken to:* This refers to comparing an image (geometric figure following a transformation) to its preimage (figure prior to the transformation).
- scale factor: The amount by which the image grows (dilates) or shrinks (reduces) is called the "Scale Factor". The general formula for dilating a point with coordinates of (2,4) by a scale factor of ½ is: (2 ½, 4 ½) or (1, 2); scale factor of 1 produces a congruent figure: (2 1, 4 1) or (2, 4).



rigid: The figures must keep the same size and shape.

non-rigid: The figures that change size but not shape.

transformation notation:

Preimage is the figure prior to the transformation. (A, B, C) **Image** is the figure after the transformation. (A', B', C') A', B', C 'are called A prime, B prime, and C prime. $A \rightarrow A' \quad B \rightarrow B' \quad C \rightarrow C'$

Angles are taken to angles and lines are taken to lines in this picture.



- *similar:* Two polygons are similar polygons if corresponding angles have the same measure and corresponding sides are in proportion. We say that there is *similarity* between similar figures if the two facts are true.
- *congruent:* Figures that have the same shape and size are congruent. Sides are congruent if they are the same length. Angles are congruent if they have the same number of degrees.



angle sum:

Draw a triangle ABC and cut out the three angles.



Rearrange the three angles to form a straight angle on a straight line. The angle sum of a triangle is 180° because $m\angle A + m\angle B + m\angle C = 180^\circ$ shown in the diagram below.



exterior angle of triangles:



Solution:

 $x + 58^\circ = 180^\circ$ {Sum of adjacent angles forming a straight line} $x + 58^\circ - 58^\circ = 180^\circ - 58^\circ$ (Subtract 58° from both sides) $x = 122^\circ$

Also, $y+60^\circ+58^\circ=180^\circ$ (Angle sum of a triangle) $y+118^\circ=180^\circ$ (Subtract 118° from both sides) $y+118^\circ-118^\circ=180^\circ-118^\circ$ $y=62^\circ$

Thus, $x = 122^{\circ}, y = 62^{\circ}$. $\angle B + \angle y = 122^{\circ}$ and $\angle x = 122^{\circ}$ Therefore $\angle B + \angle y = \angle x$ $60^{\circ} + 62^{\circ} = 122^{\circ}$ $60^{\circ} + 62^{\circ} = 122^{\circ}$

The measure of an exterior angle at a vertex of a triangle equals the sum of the measures of the interior angles at the other two vertices of the triangle.

parallel lines cut by a transversal:



Line *r* and line *s* are parallel.

Line *t* is the transversal.

 $\angle 1 = \angle 5$, $\angle 4 = \angle 8$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$. They are corresponding angles.

 $\angle 4 = \angle 6$ and $\angle 3 = \angle 5$. They are alternate interior angles.

 $\angle 1 = \angle 7$ and $\angle 2 = \angle 8$. They are alternate exterior angles.

angle-angle criterion for similarity of triangles: If two angles of one triangle are congruent to two angle of another triangle then the triangles are similar.





 $\angle B \cong \angle E$

 $\triangle ABC \sim \triangle DEF$

proof of the Pythagorean Theorem and it converthe legs equals the square of the hypotenuse shows the parts of a right triangle.

squares of below



$$leg^2 + leg^2 = hypotenuse^2$$
hypotenuse^2 - leg^2 = leg^2 $3^2 + 4^2 = x^2$ $26^2 - 24^2 = x^2$ $9 + 16 = x^2$ $676 - 576 = x^2$ $25 = x^2$ $100 = x^2$ $\sqrt{25} = x$ $\sqrt{100} = x$ $5 = x$ $10 = x$

distance formula: The distance 'd' between the points $A = (x_1, y_1)$ and $B = (x_2, y_2)$ is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance formula can be obtained by creating a triangle and using the Pythagorean Theorem to find the length of the hypotenuse. The hypotenuse of the triangle will be the distance between the two points.

This formula is an application of the Pythagorean Theorem for right triangles:



volume of cones:



The formula for the volume of a cone can be determined from the volume formula for a cylinder. We must start with a cylinder and a cone that have equal heights and radii, as in the diagram below.



Imagine copying the cone so that we had three congruent cones, all having the same height and radii of a cylinder. Next, we could fill the cones with water. As our last step in this demonstration, we could then dump the water from the cones into the cylinder. If such an experiment were to be performed, we would find that the water level of the cylinder would perfectly fill the cylinder.



This means it takes the volume of three cones to equal one cylinder. Looking at this in reverse, each cone is one-third the volume of a cylinder. Since a cylinder's volume formula is V = Bh, then the volume of a cone is one-third that formula, or $V = \frac{Bh}{3}$. Specifically, the cylinder's volume formula is $V = \pi r^2 h$ and the cone's volume formula is $V = \frac{\pi r^2 h}{2}$.

volume of cylinders: The process for understanding and calculating the volume of cylinders is identical to that of prisms, even though cylinders are curved. Here is a general cylinder.



Let's start with a cylinder of radius 3 units and height 4 units.



We fill the bottom of the cylinder with unit cubes. This means the bottom of the prism will act as a container and will hold as many cubes as possible without stacking them on top of each other. This is what it would look like.



The diagram above is strange looking because we are trying to stack cubes within a curved space. Some cubes have to be shaved so as to allow them to fit inside. Also, the cubes do not yet represent the total volume. It only represents a partial volume, but we need to count these cubes to arrive at the total volume. To count these full and partial cubes, we will use the formula for the area of a circle.



The radius of the circular base (bottom) is 3 units and the formula for area of a circle is $A = \pi r^2$. So, the number of cubes is $(3.14)(3)^2 = (3.14)(9)$, which to the nearest tenth, is equal to 28.3.

If we imagine the cylinder like a building (like we did for prisms above), we could stack cubes on top of each other until the cylinder is completely filled. It would be filled so that all cubes are touching each other such that no space existed between cubes. It would look like this.



To count all the cubes above, we will use the consistency of the solid to our advantage. We already know there are 28.3 cubes on the bottom level and all levels contain the exact number of cubes. Therefore, we need only take that bottom total of 28.3 and multiply it by 4 because there are four levels to the cylinder. $28.3 \times 4 = 113.2$ total cubes to our original cylinder.

If we review our calculations, we find that the total bottom layer of cubes was found by using the area of a circle, πr^2 . Then, we took the result and multiplied it by the cylinder's height. So the volume of a cylinder is π times the square of its radius times its height

volume of spheres: A sphere is the locus of all points in a region that are equidistant from a point. The two-dimensional rendition of the solid is represented below.



To calculate the surface area of a sphere, we must imagine the sphere as an infinite number of pyramids whose bases rest on the surface of the sphere and extend to the sphere's center. Therefore, the radius of the sphere would be the height of each pyramid. One such pyramid is depicted below.



The volume of the sphere would then be the sum of the volumes of all the pyramids. To calculate this, we would use the formula for volume of a pyramid, namely $V = \frac{Bh}{3}$. We would take the sum of all the pyramid bases, multiply by their height, and divide by 3.

First, the sum of the pyramid bases would be the <u>surface area of a sphere</u>, SA = $4\pi r^2$. Second, the height of each of the pyramids is the radius of the sphere, r. Third, we divide by three. The result of these three actions is volume of a sphere V = $\frac{(4\pi r^2)(r)}{3}$ or V = $\frac{4\pi r^3}{3}$.

- *bivariate data:* When we conduct a study that examines the relationship between two variables, we are working with bivariate data. A study that examines a potential relationship between the height and weight of high school students is based on bivariate data, or two variables (height and weight). A study that looks at only one variable, for example a survey to estimate the average height of high school students, is based on univariate data.
- *scatter plot:* Scatter plots are used to analyze patterns in bivariate data. These patterns are described in terms of linearity, slope, and strength.
 - Linearity refers to whether a data pattern is linear (straight) or nonlinear (curved).
 - Slope refers to the direction of change in variable *y* when the value of variable *x* increases. If the value of *y* also increases, the slope is positive; but if the value of *y* decreases, the slope is negative. If the data points are horizontal, the slope is 0.
 - Strength refers to the degree of "scatter" in the plot. If the data points are widely spread, the relationship between variables is weak. If the points are concentrated around a line, clustered, the relationship is strong.

Additionally, scatter plots can reveal unusual features in data sets, such as clusters, gaps, and outliers. The scatter plots below illustrate some common patterns.



The pattern in the last example (nonlinear, zero slope, weak) is the pattern that is found when two variables are not related.

clustering: Clustering occurs when data points are concentrated around a line or in specific areas of a scatter plot to show a relationship between the two variables. This scatter plot shows no clustering data and no relationship between the bivariate data.









positive association: The data on this scatter plot has a weak, linear, positive association.

The data on this scatter plot has a strong, non-linear (curved), positive association.



Negative association: The data on this scatter plot has a strong, linear, negative association



The data on this scatter plot has a strong, non-linear (curved), negative association.



linear association: The scatter plot on the left shows a positive linear association. The scatter plot in the middle shows a linear slope of 0. The scatter plot on the rights shows a negative linear association.



non-linear association: The scatter plot on the left shows a non-linear (curved), positive association. The scatter plot in the middle shows no association. The scatter plot on the rights shows a non-linear, negative association.



- *bivariate measurement data:* This refers to a relationship between two sets of data that can be evalutated by scales, as in a scatter plot.
- *bivariate categorical data:* This refers to a relationship between two sets of data that are descriptive. Analysis of categorical data generally involves the use of data tables. A two-way table presents categorical data by counting the number of observations that fall into each group for two variables, one divided into rows and the other divided into columns. For example, a survey was conducted of a group of 20 individuals who were asked to identify their hair and eye color. A two-way table presenting the results might appear as follows:

	Eye	Blue	Green	Brown	Hazel	Total
	Color					
Hair Color						
Blonde		2	1	2	1	6
Red		1	1	2	0	4
Brown		1	0	4	2	7
Black		1	0	2	0	3
Total		5	2	10	3	20

HAIR AND EYE COLOR

frequency: In a collection of data, frequency refers to the number of items in a given category.

relative frequency: The term relative frequency is used for the ratio of the observed frequency of some outcome and the total frequency of the random experiment. Suppose a random experiment is repeated *n* times and a particular outcomes is observed, then the ratio $\frac{f}{n}$ is called the relative frequency of the outcome which has been observed *f* times. An example of relative frequency is:

A factory worker selects 100 light bulbs from a certain lot of new bulbs to examine whether they are good or defective. He finds 60 bulbs that are defective. The symbol *n* represents the number of times the experiment is repeated (100 times) and the symbol *f* may be used for the observed frequency (60 bulbs). Thus the relative frequency is:

Relative frequency =
$$\frac{f}{n} = \frac{60}{100} = 0.6$$

two-way table: A two-way table presents categorical data by counting the number of observations that fall into each group for two variables, one divided into rows and the other divided into columns. For example, a survey was conducted with a group of twenty individuals who were asked to identify their hair and eye color. A two-way table presenting the results might appear as follows:

	Eye	Blue	Green	Brown	Hazel	Total		
	Color							
Hair Color								
Blonde		2	1	2	1	6		
Red		1	1	2	0	4		
Brown		1	0	4	2	7		
Black		1	0	2	0	3		
Total		5	2	10	3	20		

HAIR AND EYE COLOR

categorical variables: This refers to variables that are descriptive rather than quantitative. For example, a survey was conducted with a group of twenty individuals who were asked to identify their hair and eye color. Hair color and eye color are categorical variables. A two-way table presenting the results might appear as follows:

	Eye	Blue	Green	Brown	Hazel	Total	
	COIOI						
Hair Color							
Blonde		2	1	2	1	6	
Red		1	1	2	0	4	
Brown		1	0	4	2	7	
Black		1	0	2	0	3	
Total		5	2	10	3	20	

HAIR AND EYE COLOR